Неабелевы дуальности
в теории струн типа II

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Правила Бушера

$$S = \int d\tau d\sigma (\sqrt{-h} h^{ab} G_{\mu\nu} + \epsilon^{ab} B_{\mu\nu}) \partial_a X^{\mu} \partial_b X^{\nu}$$

$$\sigma_{\pm} = 1/2(\tau \pm \sigma)$$

$$S = \int d\tau d\sigma (G + B)_{\mu\nu} \partial_+ X^{\mu} \partial_- X^{\nu}$$

$$\theta \to \theta + \xi(\theta).$$

$$S[\theta+\xi] = \int d\tau d\sigma (G_{\theta\theta}(\partial_+\theta+\partial_+\xi)(\partial_-\theta+\partial_-\xi)+E_{\hat{\alpha}\theta}\partial_+ X^{\hat{\alpha}}(\partial_-\theta+\partial_-\xi)+$$

$$+E_{\theta\hat{\alpha}}(\partial_+\theta + \partial_+\xi)\partial_- X^{\hat{\alpha}} + E_{\hat{\alpha}\hat{\beta}}\partial_+ X^{\hat{\alpha}}\partial_- X^{\hat{\beta}}$$

$$S[\theta,\lambda]=\int d\tau d\sigma(G_{\theta\theta}D_+\theta D_-\theta+E_{\hat{\alpha}\theta}\partial_+X^{\hat{\alpha}}D_-\theta+$$

$$E_{\theta\hat{\alpha}}D_+\theta\partial_-X^{\hat{\alpha}}+E_{\hat{\alpha}\hat{\beta}}\partial_+X^{\hat{\alpha}}\partial_-X^{\hat{\beta}}+\lambda F_{+-})$$

$$F_{+-}=\partial_+A_--\partial_-A_+$$

$$A_+=\frac{1}{G_{\theta\theta}}\partial_+\lambda-\frac{E_{\hat{\alpha}\theta}}{G_{\theta\theta}}\partial_+X^{\hat{\alpha}}-\partial_+\theta$$

$$A_-=-\frac{1}{G_{\theta\theta}}\partial_-\lambda-\frac{E_{\theta\hat{\alpha}}}{G_{\theta\theta}}\partial_-X^{\hat{\alpha}}-\partial_-\theta$$

$$
\begin{aligned}
G'_{\lambda\lambda} &= \frac{1}{G_{\theta\theta}}, \\
E'_{\lambda\hat{\alpha}} &= \frac{E_{\theta\hat{\alpha}}}{G_{\theta\theta}}, \\
E'_{\hat{\alpha}\lambda} &= -\frac{E_{\hat{\alpha}\theta}}{G_{\theta\theta}}, \\
E'_{\hat{\alpha}\hat{\beta}} &= E_{\hat{\alpha}\hat{\beta}} - \frac{E_{\hat{\alpha}\theta}}{G_{\theta\theta}} E_{\theta\hat{\beta}}.
\end{aligned}
$$

Неабелева Т-дуальность

$$S = \int dx^{\tilde{\mu}}(G_{\tilde{\mu}\tilde{\nu}} * + B_{\tilde{\mu}\tilde{\nu}}) \wedge dx^{\tilde{\nu}}$$

$$= \int dx^{\mu}(G_{\mu\nu} * + B_{\mu\nu}) \wedge dx^{\nu} + 2dy^{m}(G_{m\nu} * + B_{m\nu}) \wedge dx^{\nu}$$

$$+ dy^{m}(G_{mn} * + B_{mn}) \wedge dy^{n}$$

$$g^{-1}dg = (g^{-1}dg)^{I} T_{I} = \sigma^{I}_{m} dy^{m} T_{I}$$

$$G_{mn} = \sigma^{I}_{m}\sigma^{J}_{n} G_{IJ(x)}$$

$$S = \int dx^{\mu}(G_{\mu\nu} * + B_{\mu\nu}) \wedge dx^{\nu} + 2A^{I}(G_{I\nu} * + B_{I\nu}) \wedge dx^{\nu} +$$

$$A^{I}(G_{IJ} * + B_{IJ}) \wedge A^{J} + \tilde{\lambda}_{I} F^{I},$$

$$F^{I} = 2dA^{I} + f_{JK}{}^{I} A^{J} \wedge A^{K}$$

$$G'_{\mu\nu} + B'_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu} - E_{\mu I} N_+^{IJ} E_{J\nu},$$

$$G'^{I}_{\mu} = \frac{1}{2} N_+^{IJ} E_{J\mu} - \frac{1}{2} N_+^{IJ} E_{\nu} J$$

$$B'^{I}_{\mu} = \frac{1}{2} N_+^{IJ} E_{J\mu} + \frac{1}{2} N_+^{IJ} E_{\nu} J$$

$$G'^{IJ} + B'^{IJ} = N_+^{IJ} \qquad N_+^{IJ} = G^{IJ} + B^{IJ} + \tilde{\lambda}_K f_{IJ}^K$$

Дубль Дринфельда

$$[T_a, T_b] = f_{ab}^c T_c, \qquad a, b, c... = 1, ..4$$

$$[\tilde{T}^a, \tilde{T}^b] = \tilde{f}_c^{ab} \tilde{T}^c, \qquad a, b, c... = 1, ..4$$

$$[T_A, T_B] = F_{AB}^C T_C, \qquad A, B, C... = 1, ..8$$

$$T_A = (\tilde{T}^a, T_a)$$

$$[T_a, \tilde{T}^b] = F_a^{bc} T_c + F_{ac}^b \tilde{T}^c = \tilde{f}_a^{bc} T_c - f_{ac}^b T^c$$

$$F_{abc} = 0, \qquad F_{ab}^c = f_{ab}^c, \qquad F_a^{bc} = \tilde{f}_a^{bc}, \qquad F^{abc} = 0$$

$$F_{[AB}^E F_{CD]E} = 0$$

$f_{ab}^{c} = 0$ – Абелева Т-дуальность
$f_{ab}^{c} \neq 0$, $\tilde{f}_{a}^{bc} = 0$ - Неабелева Т-дуальность
$f_{ab}^{c} \neq 0$, $\tilde{f}_{a}^{bc} \neq 0$ - Пуассон-Лиева Т-дуальность

$$2\tilde{f}_{[a}^{ed} f_{b]d}^{c} - 2\tilde{f}_{[a}^{cd} f_{b]d}^{e} - \tilde{f}_{d}^{ec} f_{ab}^{d} = 0$$

$$2f_{e[a}^{c} \tilde{f}_{b]}^{ed} - 2\tilde{f}_{[a}^{ce} f_{b]e}^{d} - f_{ab}^{e} \tilde{f}_{e}^{cd} = 0.$$

Таблица: Ненулевые струтурные константы алгебры $\tilde{g}$

$g_{4,1}$	$\tilde{f}_i^{1,2}, \tilde{f}_i^{1,3}, \tilde{f}_i^{1,4}, \tilde{f}_i^{2,3}, \tilde{f}_i^{2,4}, \tilde{f}_i^{3,4}$
$g_{4,2}$	$\tilde{f}_1^{1,2}, \tilde{f}_1^{1,3}, \tilde{f}_3^{1,2}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}, \tilde{f}_4^{2,3}, \beta \neq 0$
$g_{4,3}$	$\tilde{f}_1^{1,2}, \tilde{f}_2^{2,3}, \tilde{f}_3^{1,2}, \tilde{f}_3^{2,3}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}, \tilde{f}_4^{2,3}$
$g_{4,4}$	$\tilde{f}_2^{1,2}, \tilde{f}_1^{1,3}, \tilde{f}_3^{1,2}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}, \tilde{f}_4^{2,3}$
$g_{4,5}$	$\tilde{f}_1^{1,2}, \tilde{f}_1^{1,3}, \tilde{f}_2^{1,2}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}, \tilde{f}_4^{2,3}$

$g_{4,6}$	$\tilde{f}_1^{1,2}, \tilde{f}_1^{1,3}, \tilde{f}_2^{1,3}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}, \tilde{f}_4^{2,3}$
$g_{4,7}$	$\tilde{f}_2^{1,2}, \tilde{f}_3^{1,2}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}$
$g_{4,8}$	$\tilde{f}_2^{1,2}, \tilde{f}_3^{1,3}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}, \beta \neq 1$
$g_{4,9}$	$\tilde{f}_2^{1,3}, \tilde{f}_4^{1,4}, \tilde{f}_4^{1,2}, \tilde{f}_4^{1,3}$
$g_{4,10}$	$\tilde{f}_3^{1,3}, \tilde{f}_3^{1,2}, \tilde{f}_3^{2,3}$
$2g_{2,1}$	$\tilde{f}_2^{1,2}, \tilde{f}_2^{1,3}, \tilde{f}_4^{3,4}$

Спасибо за внимание!