Лекция 6

10

повторение

$$\mathcal{L} = \mathcal{L}_{free}[\varphi] + \lambda\varphi^4 \qquad , \mathcal{L}_{free}[\varphi] = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{m^2}{2}\varphi^2$$

$$Z = \underbrace{\int D\varphi\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_{free}[\varphi]}}_{Z_0} \cdot e^{-\int d\beta \int d^3x\, \lambda\varphi^4} =$$

$$= Z_0\left(1 - \lambda\, d^4x \langle\varphi^4(x)\rangle + \frac{\lambda^2}{2}\int d^4x\, d^4y\, \langle\varphi^4(x)\varphi^4(y)\rangle + \dots\right)$$

$3\ \infty^x$

$36\ \infty\infty \quad + 12 \quad + \infty_x\ \infty_y$

$$\underset{-\beta F}{\ln Z} = \underset{-\beta F_0}{\ln Z_0} - \lambda\int d^4x \langle\varphi^4(x)\rangle - \frac{\lambda^2}{2}\left(\int d^4x \langle\varphi^4(x)\rangle\right)^2 + \frac{\lambda^2}{2}\int d^4x \int d^4y\, \langle\varphi^4(x)\varphi^4(y)\rangle$$

несвязные диагр.

$f = \frac{F}{V}$

$\infty_x\ \infty_y$ – сокращаются.

$$\langle\varphi^{(4)}(x)\rangle = 3\, G^2(0)$$

ф-я Грина,
$$\langle\varphi(x_1)\varphi(x_2)\rangle = \cancel{\frac{1}{Z_0}\int D\varphi \left(1 - \lambda\varphi^4(x) + \dots\right) e^{\int d^4x\, \mathcal{L}_{free}}} \; \cancel{\frac{1}{Z_0}\int D\varphi\, \varphi\varphi(\ )} \Big/ \frac{Z}{Z_0}$$

$$\cancel{= G(x_1 - x_2) - \lambda\langle\varphi^4(x)\varphi(x_1)\varphi(x_2)\rangle + \lambda^2}\dots$$

$$= \frac{1}{Z}\int D\varphi\, \varphi(x_1)\varphi(x_2)\, e^{-\int d^4x\, \lambda\varphi^4(x)}\, e^{-\int d^4x\, \mathcal{L}_{free}} = \frac{\frac{1}{Z_0}\int D\varphi\dots}{Z/Z_0} =$$

$$= \cancel{\frac{1}{Z_0}\int D\varphi\, \varphi(x_1)\varphi(x_2)\, e^{-\int d^4x\, \lambda\dots}}$$

$$= G(x_1 - x_2) - \lambda \int d^4x \langle\varphi(x_1)\varphi(x_2)\varphi^4(x)\rangle + \lambda^2 \dots =$$

$$= G(x_1 - x_2) - \underset{12}{\lambda}\, G(x_1 - x)\, \underbrace{G(x - x)}_{G(0)}\, G(x - x_2) + \dots$$

$$= G(x_1 - x_2) - \overset{12}{\lambda}\, G(0)\, G(x_1 - x_2) = G(x_1 - x_2)\left(1 - \overset{12}{\lambda}\, G(0)\right)$$

$x_1 \quad x \quad x_2 \quad -12$

$$G(0) = \frac{1}{(2\pi)^3\beta}\sum_n \int \frac{d^3p}{\omega_n^2 + \vec{p}^2 + m^2} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{\vec{p}^2 + m^2}}\left[\frac{1}{2} + \frac{1}{e^{\sqrt{p^2+m^2}/T} - 1}\right]$$

сравниваем: $T = 0$, $+$ $\longrightarrow$ голубасик – бесконечная конст., кот. эго член полностью сокращается перенормировкой

$T \neq 0$ $\longrightarrow$ не полностью сокр. перенормировкой

$G(0)^{ren} \sim T^2$

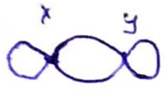
б1

Свободная энергия в 3х порядках по λ; в разложении по малому m/T

$$f_{(0)}(T) = -\frac{\pi^2 T^4}{90} + \frac{m^2 T^2}{24} - \frac{m^3 T}{12\pi} + O(m^4)$$

$$f_{(1)}(T) = \frac{3}{4}\lambda\left[\frac{T^2}{12} - \frac{mT}{4\pi} + O(m^2)\right]^2 = \frac{3}{4}\lambda\left[\frac{T^4}{144} - \frac{mT^3}{24\pi} + O(m^2T^2)\right]$$

$$f_2(T) = -\frac{9}{4}\lambda^2 \frac{T^4}{144}\,\frac{T}{8\pi m} + O(1)$$



$$= G(x-x)\,G(x-y)\,G(y-y)\,G(y-x) =$$

$$= G(0)^2 \cdot G(x-y)\,G(y-x)$$

$$= G(0)^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\omega_n^2 + k^2 + m^2)^2}$$

$m=0$
вклад $n=0$
расходится
при $k \to 0$

(с 1 стр)

$$G(p) = G_0(p) + G_0(p)\,\overbrace{(-12\lambda G(0))}^{\Pi}\,G_0(p) + \dots$$

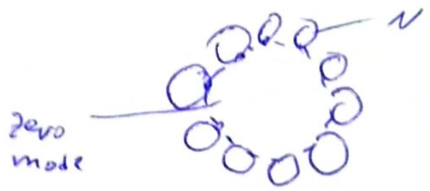
$$= G_0(p)\,(1 + \Pi G_0 p + \dots) = \frac{G_0(p)}{1 - \Pi G_0(p)} = \frac{1}{p^2 + m^2 - \Pi}$$

$$m_{eff}^2 = m^2 - \Pi = m^2 + 12\lambda G(0) = m^2 + \lambda T^2$$

$$G^{-1}(p) = G_0(p)^{-1} - \Pi$$

Инфракрасное пересуммирование диаграмм.

• Рассмотрим ИК расходящиеся диаграммы порядка N, N вкладов со всеми мацубар. модами, 1 — только с нулевой модой, вида:



$N=2$:  если диагр ∞, в средней петле 0 мода.

6 - из тензорн. вкладов тору

• $$f^{(N)}_{daisy}(\tau) = \frac{(-1)^{N+1}}{N!}\lambda^N \langle \phi\phi\phi\phi \ \phi\phi\phi\phi \ \dots \ \phi\phi\phi\phi \ \phi\phi\phi\phi \rangle =$$

$$2(N-1)\times 2(N-2)\times \dots 2\cdot 2$$

$$= \frac{(-1)^{N+1}}{N!}\lambda^n \, 6^N \cdot 2^{N-1}(N-1)!\,[G(0)]^N \cdot T\int \frac{d^3p}{(2\pi)^3}\left(\frac{1}{p^2+m^2}\right)^N$$

• $N=1$ $$\int \frac{1}{p^2+m^2}\frac{d^3p}{(2\pi)^3} = \int_0^\Lambda \frac{p^2dp}{2\pi^2}\frac{1}{p^2+m^2} = \frac{m}{2\pi^2}\int_0^\Lambda \frac{x^2dx}{x^2+1} = \frac{m}{2\pi^2}\left(\frac{\Lambda_{UV}}{m}-\frac{\pi}{2}\right) =$$

$$= \frac{\Lambda_{UV}}{2\pi^2} - \frac{m}{4\pi} = \quad \to \text{убирается УФ-перенормировкой}$$

$$= \frac{\partial}{\partial m^2}\left(-\frac{m^3}{6\pi}\right)$$

УФ расх. уже нет

• $N=2$ $$\int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+m^2)^2} = -\frac{\partial}{\partial m^2}\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2+m^2} = -\left(\frac{\partial}{\partial m^2}\right)^2\left(-\frac{m^3}{6\pi}\right)$$

• N $$\int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+m^2)^N} = \frac{(-1)^N}{(N-1)!}\left(\frac{d}{dm^2}\right)^N\left(\frac{m^3}{6\pi}\right)$$

• $$f^{(N)}_{daisy}(\tau) = \frac{(-1)^{N+1}}{N!}\lambda^N\cdot 6^N\, 2^{N-1}(N-1)!\left(\frac{T^2}{12}\right)^N T\,\frac{(-1)^N}{(N-1)!}\left(\frac{d}{dm^2}\right)^N\left(\frac{m^3}{6\pi}\right) =$$

$$= -\frac{T}{2}\frac{1}{N!}(\lambda T^2)^N\left(\frac{d}{dm^2}\right)^N\left(m^3/6\pi\right)$$

суммирование daisy диаграмм

$$\sum_{N=2}^{\infty} \frac{1}{N!} \left(\lambda T^2\right)^N \left(\frac{d}{dm^2}\right)^N \left(-\frac{m^3 T}{12\pi}\right) =$$

ряд тейлора $(a+x)^{3/2}$, $a = \lambda T^2$, $x = m^2$

разложение по a

$$= -\frac{T}{12\pi}\left[\left(m^2+\lambda T^2\right)^{3/2} - \frac{3}{2} m\right] = f^{(T)}_{\sum_{N=2}^{\infty} daisy(m)}$$

$m \to 0$

$$f^{(T)}_{daisy} \to -\frac{T}{12\pi}\cdot\lambda^{3/2}T^{3/2}$$

$$f(T) = -\frac{\pi^2 T^4}{90} + \frac{\lambda T^4}{48} - \frac{\lambda^{3/2} T^4}{12\pi} + \dots$$

$$= -\frac{\pi^2 T^4}{90}\left(1 - \frac{15}{4}\left(\frac{\lambda}{\pi^2}\right) + \frac{15}{2}\left(\frac{\lambda}{\pi^2}\right)^{3/2} + \dots\right)$$

II.

$$\mathcal{L}_E = \underbrace{\frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2} m_{eff}^2 \varphi^2_{n=0}}_{\mathcal{L}_0} \underbrace{+\frac{3}{4}\lambda\varphi^4 - \frac{1}{2} m_{eff}^2\varphi^2_{n=0}}_{\mathcal{L}_{int}}$$

$$m_{eff}^2 = 3\lambda G(0) = 3\lambda\cdot\frac{T^2}{12} = \frac{\lambda T^2}{4}$$

$m_{eff}^2 \ll \omega_n^2 \sim T^2$

значимый вклад только в нулевую моду.

$$f_0(T) = -\frac{\pi^2 T^4}{90} + \underbrace{\frac{T}{2}\int dm^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{p^2+m_{eff}^2}}_{-m_{eff}^3 T/12\pi}$$

днно

$$f_1(T) = 3\lambda\Big[\underbrace{G'(0)}_{T^2/12} + \underbrace{\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2+m_{eff}^2}}_{-\frac{m_{eff}T}{4\pi}}\Big]^2 - \frac{1}{2} m_{eff}^2\left(-\frac{m_{eff}\cdot T}{4\pi}\right) =$$

$$= 3\lambda\left[\frac{T^4}{144} - \frac{m_{eff}T^3}{24\pi} + \frac{m_{eff}^2 T^2}{16\pi^2}\right] + \frac{1}{8} m_{eff}^3 T = \frac{\lambda T^4}{48} - \lambda\frac{m_{eff}T^3}{8\pi} + \frac{3\lambda m_{eff}^2 T^2}{16\pi^2} + \frac{m_{eff}^3 T}{8}$$

$$f(T) = -\frac{\pi^2 T^4}{90} - \frac{\lambda^{3/2}T^4}{12\pi} - \frac{\lambda^{3/2}T^4}{8\pi} + \frac{3\lambda^3 T^4}{16\pi^2} + \frac{\lambda^{3/2}T^4}{8\pi}$$

$$\Pi = 12\lambda \int \frac{d^3p}{(2\pi)^3}\, T \sum_n \frac{1}{\omega_n^2 + p^2 + \Pi} = 12\lambda \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{p^2+\Pi}} \frac{1}{e^{\sqrt{p^2+\Pi}/T} - 1} \stackrel{\circ}{=}$$

$$\cancel{\left\| p^2 + \Pi = x \right\| = 12\lambda \int_\Pi^\infty \frac{\sqrt{x-\Pi}\, dx}{4\pi^2\, x} \frac{1}{e^{x/T}-1} = \frac{3\lambda}{\pi^2}}$$

$$\cancel{p = \sqrt{x-\Pi}}$$

$2p\,dp = dx$

$$x = \sqrt{p^2/\Pi + 1} \qquad \Pi x^2 = p^2 + \Pi \qquad p^2 = \Pi(x^2-1)$$

$$p\,dp = \Pi x\, dx$$

$$\sqrt{p^2+\Pi} = \sqrt{\Pi}\, x$$

$$\stackrel{\circ}{=} \frac{12\lambda}{2\pi^2} \int_1^\infty \frac{\sqrt{\Pi}\sqrt{x^2-1}\cdot \Pi x\, dx}{\sqrt{\Pi}\, x} \frac{1}{e^{x\sqrt{\Pi}/T} - 1} = \Pi \frac{6\lambda}{\pi^2} \int_1^\infty \frac{\sqrt{x^2-1}\, dx}{e^{x\Pi^{1/2}/T} - 1}$$

$$1 = \frac{6\lambda}{\pi^2} \int_0^\infty \frac{\sqrt{x^2-1}\, dx}{e^{x\Pi^{1/2}/T} - 1} = \frac{6\lambda}{\pi^2} \frac{2\pi^2}{\Pi/T^2} \left[\frac{1}{12} - \frac{\Pi^{1/2}}{4\pi T} + O\left(\frac{\Pi}{T^2} \ln \Pi/T^2\right) \right]$$

$$\frac{\Pi}{T^2} = 12\lambda \left[\frac{1}{12} - \frac{\Pi^{1/2}}{4\pi T} + \dots \right]$$

$$\Pi = \lambda T^2 - \frac{3\lambda \Pi^{1/2}}{\pi T} \dots$$

$$\Pi = \lambda T^2 - \frac{3\lambda^{3/2} T^3}{\pi} + \dots$$