$$T\sum_n \frac{1}{\omega_n^2+\vec{p}^{\,2}+m^2} = \frac{1}{\sqrt{\vec{p}^{\,2}+m^2}}\left[\frac{1}{2}+\frac{1}{e^{\sqrt{\vec{p}^{\,2}+m^2}/T}-1}\right]$$

n'

$-$ $+$

$$f_b^{(T)} = T\int\frac{d^3p}{(2\pi)^3}\ln\left(1-e^{-\sqrt{\vec{p}^{\,2}+m^2}/T}\right)$$

$$f_f^{(T)} = -4T\int\frac{d^3p}{(2\pi)^3}\ln\left(1+e^{-\sqrt{\vec{p}^{\,2}+m^2}/T}\right)$$

$\not{\partial}^3$

$$Z[J] = \int\limits_{\substack{p.b.c.\\ \varphi(0)=\varphi(\beta)}} D\varphi \exp\left[-\int_0^\beta d\tau \int d^3x \, \frac{1}{2}\varphi[-\Box+m^2]\varphi + \int_0^\beta d\tau \int d^3x \, J(x,\tau)\varphi(x,\tau)\right]$$

$$\langle \varphi(x,\tau)\varphi(y,\tau')\rangle = \frac{1}{Z}\frac{\delta}{\delta J(x,\tau)}\frac{\delta}{\delta J(y,\tau')} Z[J]\Big|_{J=0}$$

$$\langle \varphi(x_1,\tau_1)\dots\varphi(x_n,\tau_n)\rangle = \frac{1}{Z}\frac{\delta}{\delta J(x_1,\tau_1)}\dots\frac{\delta}{\delta J(x_n,\tau_n)} Z[J]\Big|_{J=0}$$

берем инт. по φ - квадратичный оператор, обратный $(-\Box+m^2)$

$$G(x-y) = \frac{1}{(2\pi)^3\beta}\sum_n \int d^3p \, \exp[i\omega_n(\tau_x-\tau_y) + i\vec{p}(x-y)] \cdot G(p)$$

$$G(p) = \frac{1}{\omega_n^2 + \vec{p}^{\,2} + m^2}$$

$$Z[J] = Z_\beta \exp\left[\int_0^\beta d\tau_x \, d^3x \int_0^\beta d\tau_y \, dy \, \frac{1}{2} J(x,\tau_x) G(x-y) J(y,\tau_y)\right]$$

$$\langle \varphi(x,\tau_x)\varphi(y,\tau_y)\rangle = G(x-y)$$

$$\langle \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle = \frac{\delta^3}{\delta J_1 \delta J_2 \delta J_3}\left(J(z)G(z-x_4) \cdot Z[J]\right)\Big|_{J=0} =$$

$$= \frac{\delta^2}{\delta J_1 \delta J_2}\left[\left(G(x_3-x_4) + J(z)G(z-x_4)J(w)G(w-x_3)\right)Z[J]\right]\Big|_{J=0} =$$

$$= \frac{\delta}{\delta J_1}\Big[G(x_3-x_4)\cdot J(z)G(z-x_2) + J(x_2)G(x_2-x_4)J(w)G(w-x_3) +$$
$$+ J(z)G(z-x_4)J(x_2)G(x_2-x_3)\Big] =$$

$$= G(x_3-x_4)G(x_1-x_2) + G(x_2-x_4)G(x_1-x_3) + G(x_1-x_4)G(x_2-x_3)$$

4х точечная ф-я Грина ~~ра~~ для свободной теории распалась на 2х точечные — получили суть теоремы Вика.

$$\langle \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle$$

Взаимодействующая теория.

$$\mathcal{L} \mapsto \mathcal{L} + \frac{\lambda}{4!}\varphi^4$$

$$Z[J] = \int_{p.b.c.} D\varphi \exp\left[-\int d\tau \int d^3x \left[\tfrac{1}{2}\varphi[-\Box + m^2]\varphi + J(x)\varphi(x)\right]\right] e^{-\int d^4x \frac{\lambda}{4!}\varphi^4(x)} =$$

$$= \int_{p.b.c.} D\varphi \left[1 - \frac{\lambda}{4!}\int d^4z\, \varphi^4(z) + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 \int d^4z\, \varphi^4(z) \int d^4w\, \varphi^4(w) \pm \dots\right] \exp[\ ]$$

$$= Z_0 - \frac{\lambda}{4!}\int d^4w \langle \varphi(w)\varphi(w)\varphi(w)\varphi(w)\rangle$$

$$Z[J] = Z_0[J] - \frac{\lambda}{4!}\int d^4z \frac{\delta^4 Z_0[J]}{\delta J(z)^4} + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 \int d^4z \int d^4w \frac{\delta^8 Z_0[J]}{\delta J(z)^4 \delta J(w)^4}$$

$$Z[J=0] = \exp[-\beta F]$$

$$\beta F = -\ln Z = -\ln\left[Z_0 - \frac{\lambda}{4!}\int d^4z \frac{\delta^4 Z_0[J]}{\delta J(z)^4}\bigg|_{J=0} + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 \int d^4z \int d^4w \frac{\delta^8 Z_0}{\delta J_z^4 \delta J_w^4}\right]$$

$$= \underbrace{-\beta \ln Z_0}_{F_0} - \ln\underbrace{\left[1 - \frac{1}{Z_0}\frac{\lambda}{4!}\int d^4w \frac{\delta^4 Z_0[J]}{\delta J(w)^4} + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 \int d^4w \int d^4v \dots\right]}_{\text{разложение по } \lambda}$$

$$\ln(1+x) \simeq x - \frac{x^2}{2}$$

$$\beta F = \beta F_0 - \left[-\frac{1}{Z_0}\frac{\lambda}{4!}\int d^4w \frac{\delta^4 Z_0[J]}{\delta J(w)} + \frac{1}{2 Z_0}\left(\frac{\lambda}{4!}\right)^2 \int d^4w \int d^4v \frac{\delta^8 Z_0}{\delta J_w \delta J_v} - \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 \left(\frac{1}{Z_0}\int d^4w \frac{\delta^4 Z_0}{\delta J_w}\right)^2\right.$$

$$\frac{1}{Z_0}\int d^4w \frac{\delta^4 Z_0[J]}{\delta J(w)^4}\bigg|_{J=0} = \langle \varphi(w)\varphi(w)\varphi(w)\varphi(w)\rangle = \frac{\delta^3}{\delta J^3}\left(J(z)\,G(z-w)\,Z\right)\dots$$

$$= \int 3\, G(0)^2\, d^4w$$

∞^4

$$~~G(0) = \frac{1}{(2\pi)^3 \beta}\sum_n \int d^3p\, G_n(p)~~$$

$$~~3\int d^4w\, G(0)^2 = \frac{3}{(2\pi)^6 \beta^2}\cdot\beta\cdot V(\ \ )~~$$

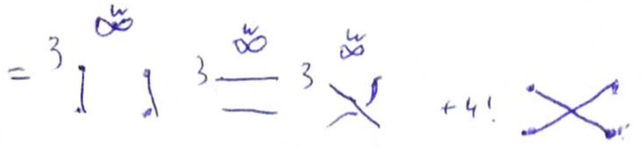
$$G(0) = \frac{1}{(2\pi)^3\beta}\sum_n \int \frac{d^3p}{\omega_n^2 + p^2 + m^2} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{p^2+m^2}}\left[\frac{1}{2} + \frac{1}{e^{\sqrt{p^2+m^2}/T} - 1}\right]$$

при $m=0$ $\quad \int^\infty \frac{p^2dp}{\dots}\frac{1}{e^{p/T}-1} = \frac{T^2}{2\pi^2}\int^\infty \frac{x\,dx}{e^x - 1} = T^2/12$ ($\int = \pi^2/6$)

$$\langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle = \frac{1}{Z_\varphi}\frac{\delta^4 Z[J]}{\delta J(x_1)\ldots\delta J(x_4)}\Big|_{J=0} = \langle 1234\rangle_0 -$$

$$-\frac{\lambda}{4!}\int d^4w\, \underbrace{\frac{\delta^8 Z_0[J]}{\delta J(x_1)\ldots\delta J(x_4)\,\delta J(w)^4}}$$

$$= -\frac{\lambda}{4!}\int d^4w\, \langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\varphi(w)\varphi(w)\varphi(w)\varphi(w)\rangle$$



$$= \langle 1234\rangle_0\left(1-\frac{\lambda}{4!}\int d^4w\,\langle\varphi(w)^4\rangle\right) +$$

$$+\frac{\lambda}{4!}\int d^4w\; G_{x_1w}\,G_{x_2w}\,G_{x_3w}\,G_{x_4w}$$

$$\langle\varphi(x_1)\varphi(x_2)\rangle = \frac{1}{Z}\frac{\delta^2 Z[J]}{\delta J(x_1)\,\delta J(x_2)}\Big|_{J=0} = G(x_1-x_2) -$$

$$-\frac{\lambda}{4!}\int d^4w\,\frac{\delta^6 Z_0[J]}{\delta J(x_1)\,\delta J(x_2)\,\delta J(w)^4} = \frac{\lambda}{4!}$$

$$= G(x_1-x_2)\left(1-\frac{\lambda}{4!}\int d^4w\,\langle\varphi(w)^4\rangle\right) - \frac{\lambda}{4!}\int d^4w\; G(x_1-w)\,G(w-w)\,G(w-x_2) =$$

$$= G(x_1-x_2)\left(1-\frac{\lambda}{4!}\int d^4w\,\langle\varphi(w)^4\rangle\right) - \frac{\lambda}{4!}\,G(0)\,G(x_1-x_2)$$

После перенормировки:

$$\beta F_{\lambda} = \beta F_0 + \frac{1}{Z_0}\lambda\int d^4w\,\underbrace{\frac{\delta^4 Z_0[J]}{\delta J(w)^4}\Big|_{J=0}} = 3\lambda\int G(0)^2\, d^4w =$$

$$f = \frac{F}{V} \qquad = 3\lambda\cdot\beta\cdot V\cdot T^4/12^2$$

$$\beta f = \beta f_0 + \beta\cdot\frac{T^4}{48}$$

$$p = p_0 - \frac{T^4}{48} = \frac{\pi^2 T^4}{90}\left(1-\frac{15\lambda}{8\pi^2}+\ldots\right)$$

$$\frac{1}{p^2+m^2}\left(1-\lambda\frac{G(0)}{p^2+m^2}+\lambda\ldots\right) = \frac{1}{p^2+m^2\left(1+\frac{\lambda G(0)}{p^2+m^2}\right)}$$

$$= \frac{1}{p^2+m^2+\lambda G(0)}$$

$$m^2 \to m^2 + \lambda\ldots$$