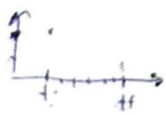
Лекция 3

$$\hat{U}(t_f, t_i) = e^{-i\hat{H}(t_f - t_i)}$$



$$\langle x_f | \hat{U}(t_f, t_i) | x_i \rangle = \int_{x[t_i]=x_i}^{x[t_f]=x_f} Dx \, e^{i\int_{t_i}^{t_f} dt\, \mathcal{L}[x]}, \qquad \mathcal{L} = \dot{x}p - H, \quad \dot{x} = \frac{\partial H}{\partial p}$$

в интеграл вкладывают траектории с гран. условием

$$\hat{\rho}_\beta = e^{-\hat{H}\beta}, \quad \beta = 1/T$$

$$Z = \mathrm{Tr}\,\hat{\rho} \equiv \int dx \langle x | e^{-H\beta} | x \rangle = \int dy \int_{x(0)=y}^{x(\beta)=y} Dx\, e^{-\int_0^\beta d\tau\, \mathcal{L}_E[x]}$$

QM → field theory

$t \to -i\tau$

$\mathcal{L} \to i\mathcal{L}$

$\mathcal{L} = T - U; \; \mathcal{L}_E = T + U$

$$X \to \varphi(x)$$

вместо значения координаты частицы

значение поля φ во всём пространстве, в завис. от x

$$\mathcal{L}[x] \mapsto \int d^3x\, \mathcal{L}[\varphi]$$

$$\langle \varphi_B(x) | U(t_f, t_i) | \varphi_A(x) \rangle = \int_{\varphi(t_A)=\varphi_A}^{\varphi(t_B)=\varphi_B} D\varphi\, e^{+i\int_{t_i}^{t_f} dt \int d^3x\, \mathcal{L}[\varphi]}$$

$U(t_f,t_i) = e^{-i\hat{H}(t_f - t_i)}$

$\mathcal{L}[\varphi] = \frac{1}{2}(\partial_t\varphi)^2 - \frac{m^2\varphi^2}{2} - \frac{1}{2}(\partial_i\varphi)^2$

$\varphi_A, \varphi_B = \Omega$

$$\mathrm{Tr}\,\hat{\rho} = \int D\varphi(x) \langle \varphi(x) | e^{-\hat{H}\beta} | \varphi(x) \rangle = \int_{\varphi(0)=\varphi(\beta)} D\varphi\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_E[\varphi]}$$

$\mathcal{L}_E[\varphi] = \frac{1}{2}(\partial_\tau\varphi)^2 + \frac{1}{2}(\partial_i\varphi)^2 + \frac{m^2\varphi^2}{2}$

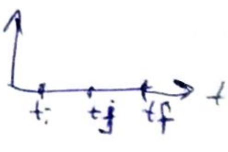
Мы хотим считать в т.п. средние от операторов между обкладками.

$$\langle \Omega, t_f | T(\hat{\varphi}(x_1, t_1) \dots \hat{\varphi}(x_n, t_n)) | \Omega, t_i \rangle$$

$$\langle \hat{A} \rangle = \frac{\mathrm{Tr}[\hat{A} e^{-\hat{H}\beta}]}{\mathrm{Tr}[e^{-\hat{H}\beta}]};$$

$\hat{A}$ в том числе составной оператор, как в QM, так и в теории поля.

$$\langle \Omega, t_f | \hat{\varphi}(x_j, t_j) | \Omega, t_i \rangle = //$$



$$= \int D\varphi_{t_j} \langle \Omega, t_f | \varphi(x_j, t_j) \rangle\, \hat{\varphi} \langle \varphi_{t_j} | \hat{\varphi}(x_j, t_j) | \Omega(t_i) \rangle =$$

$$\int D\varphi_j(x_j)\,\langle \Omega, t_f \mid \varphi_j(x_j,t_j)\rangle\,\varphi(x_j,t_j)\,\langle \varphi_j(x_j,t_j) \mid \Omega, t_i\rangle$$

// используем ф-лу эволюции $\varphi_A(x) \to \varphi_B(x)$ //

$$= \int D\varphi_j(x_j) \int_{\varphi(t_j)=\varphi_j}^{\varphi(t_f)=\Omega} D\varphi\, e^{i\int_{t_j}^{t_f} dt \int d^3x\, \mathcal{L}[\varphi]}\ \varphi(x_j,t_j) \int_{\varphi(t_i)=\Omega}^{\varphi(t_j)=\varphi_j} D\varphi\, e^{i\int_{t_i}^{t_j} dt \int d^3x\, \mathcal{L}[\varphi]} =$$

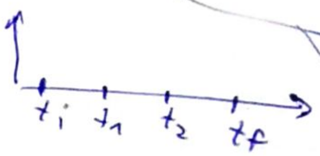
$$= \int_{\varphi(t_i)=\Omega}^{\varphi(t_f)=\Omega} D\varphi(x_j,t)\,\varphi(x_j,t_j)\, e^{i\int_{t_i}^{t_f} dt \int d^3x\, \mathcal{L}[\varphi]}$$

обратно так же, $\int D\varphi\, \varphi(x_j,t_j)\, e^{iS}$ = // разбить на 2 временных отрезка // =

$= \langle \Omega | \hat\varphi(x_j,t_j) | \Omega\rangle$ — выделим из интеграла $D\varphi(x_j,t_j)$

$$\int_{\varphi(t_i)=\Omega}^{\varphi(t_f)=\Omega} D\varphi\, \varphi(x_1,t_1)\,\varphi(x_2,t_2)\, e^{iS} =$$

фикс. t_1, t_2 есть какая-то иерархия, пусть $t_1 < t_2$



$$= \langle \Omega | \hat\varphi(x_2,t_2)\,\hat\varphi(x_1,t_1) | \Omega\rangle$$

при $t_2 > t_1$.

при $t_1 > t_2$ $\langle \Omega | \hat\varphi(x_1,t_1)\,\hat\varphi(x_2,t_2) | \Omega\rangle$

при $\forall\, t_1, t_2$ T - произведение.

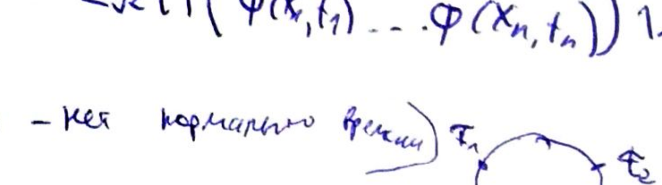
$$\int D\varphi\, \varphi(x_1,t_1)\,\varphi(x_2,t_2)\, e^{iS} = \langle \Omega | T\big(\hat\varphi(x_1,t_1)\,\hat\varphi(x_2,t_2)\big) | \Omega\rangle$$

N операторов поля.

$$\int D\varphi\, \varphi(x_1,t_1)\ldots\varphi(x_n,t_n)\, e^{iS} = \langle \Omega | T\big(\hat\varphi(x_1,t_1)\ldots\hat\varphi(x_n,t_n)\big) | \Omega\rangle$$

для термальной теории поля – нет нормального времени

$$\langle \hat\varphi(x_1)\rangle = \int_{\text{р.ус.}} D\varphi\, \varphi(x_1,\tau=0)\, e^{-S_E}$$



наблюдаемые ... величины $\to \tau=0$. | но в качестве промежуточных состояний ... теории ... $\varphi(x,\tau)$

Производящий функционал.

$$Z[J] = \int D\varphi \exp\left[iS[\varphi] + i\int d^4x\, J(x)\varphi(x) \right]$$

$$Z[0] = \int D\varphi\, e[iS[\varphi]] = \langle \Omega, t_f | \Omega, t_i \rangle$$

$J(x)$ – источник поля $\varphi(x)$

$$\frac{\delta}{\delta J(x_1)} \int d^4x\, J(x)\varphi(x) = \varphi(x_1)$$

$$\frac{1}{Z[0]} \cdot \frac{\delta}{i\delta J(x_1)} Z \Big|_{J=0} = \langle \Omega | \hat{\varphi}(x_1) | \Omega \rangle$$

$$\frac{1}{Z[0]} \left(\frac{\delta}{i\delta J(x_1)} \right) \dots \left(\frac{\delta}{i\delta J(x_n)} \right) Z \Big|_{J=0} = \langle \Omega | T(\hat{\varphi}(x_1) \dots \hat{\varphi}(x_n)) | \Omega \rangle$$

Терм. теория.

$$Z[J] = \int_{p.b.c.} D\varphi \exp\left[-S_E[\varphi] - \int d^3x\, \varphi(x) J(x) \right]$$

?

Свободное скалярное поле.
Пропагатор – 2-х точечная ф-я Грина.

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2\varphi^2}{2} \right) =$$

$$= \int d^4x \left(-\frac{1}{2}\varphi(\Box + m^2)\varphi \right)$$

$$Z[J] = \int D\varphi \exp\left[i\int d^4x \left(-\frac{1}{2}\varphi(\Box+m^2)\varphi + J(x)\varphi(x) \right)\right]$$

квадратичный по φ, возьмем как обобщение Гаусс. интеграла.

$$\int_{-\infty}^{\infty} d\vec{\varphi} \exp\left[-\frac{1}{2}\vec{\varphi}A\vec{\varphi} + \vec{J}\vec{\varphi}\right] = \sqrt{\frac{(2\pi)^n}{\det A}} \exp\left[\frac{1}{2}\vec{J}A^{-1}\vec{J}\right]$$

n степеней свободы (дискретизация) $\to \infty$

$$\hat{A} = i(\Box + m^2)$$

Обратный оператор $\Pi(x-y)$

$$i(\Box_x + m^2)\Pi(x-y) = \delta(x-y)$$

Фурье: лев. и прав. части

$$\int \frac{d^4p}{(2\pi)^4} i(\Box_x+m^2)\Pi(p)e^{ip(x-y)} = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)}$$

$$i(-p^2+m^2)\Pi(p) = 1$$

$$\Pi(p) = \frac{i}{p^2-m^2}$$ – пропагатор.

$$\Pi(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{ip(x-y)}$$

$$Z[J] = \exp\left[-\int d^4x \int d^4y \frac{1}{2} J(x)\Pi(x-y)J(y)\right]$$

$$\langle 0|T(\hat{\varphi}(x)\hat{\varphi}(y))|0\rangle = \frac{1}{Z[0]} \left.\frac{\delta^2 Z[J]}{i\delta J(x)\, i\delta J(y)}\right|_{J=0} = \Pi(x-y)$$

$$\boxed{5}$$

$$S_E = \int_0^\beta d\tau \int d^3x \left(\frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi + \frac{m^2}{2} \varphi^2 \right) =$$

$$= \int_0^\beta d\tau \int d^3x \left[\frac{1}{2} \varphi \left(-\Box + m^2 \right) \varphi \right]$$

$$Z[J] = \int D\varphi \exp \left[-\int_0^\beta d\tau \int d^3x \, \frac{1}{2} \varphi (-\Box + m^2) \varphi + \int_0^\beta d\tau \int d^3x \, J(x_\mu) \varphi(x_\mu) \right]$$

$$\hat{A} = -\Box + m^2$$

$$(-\Box + m^2)\, \Pi(x-y) = \delta^{(4)}(x-y)$$

$$\delta^{(4)}(x-y) = \frac{1}{(2\pi)^3 \beta} \sum_n \int d^3p \; e^{i\omega_n(\tau_x - \tau_y) + i\vec{p}(\vec{x}-\vec{y})}$$

разные τ и прост. части.
многомерн. Фурье по прост.,
ряд Фурье по τ.

$$\boxed{\omega_n = \frac{2\pi n}{\beta}}$$

$$(\omega_n^2 + \vec{p}^{\,2} + m^2)\, \Pi(p) = 1$$

$$\cancel{\omega_n \to \omega_n + 2\pi}$$

$$\Pi(p) = \frac{1}{\omega_n^2 + \vec{p}^{\,2} + m^2}$$

$$\tau_x \to \tau_x + \beta$$

$$\cancel{e^{i\omega_n + 2\pi i}}$$

$$e^{i 2\pi n}$$

$$\Pi(x-y) = \frac{1}{(2\pi)^3 \beta} \sum_{n \in \mathbb{Z}} \int d^3p \, \frac{1}{\omega_n^2 + \vec{p}^{\,2} + m^2} \, e^{i\omega_n(\tau_x - \tau_y) + i\vec{p}(\vec{x}-\vec{y})}$$

Кв. механика. $n = 0, 1, 2, \dots$

фермионы. $n = 0, 1$

$$\hat H = \frac{\hbar\omega}{2}(b^+ b - b b^+) \qquad [b, b^+]_+ = 1$$

Грассмановы переменные $\eta^2 = \eta^{*2} = 0$, $\eta\eta^* = -\eta^*\eta$

$\int d\eta = \int d\eta^* = 0$ $\qquad \int d\eta\,\eta = \int d\eta^*\,\eta^* = 1$

Состояния $|\eta\rangle = e^{-\eta b^+} = (1 - \eta b^+)|0\rangle = |0\rangle - \eta|1\rangle$

$b|\eta\rangle = \eta|0\rangle = \eta|\eta\rangle$

$\langle\eta| = \langle 0|e^{-b\eta^*} = \langle 0| - \langle 1|\eta^*$

$\langle\eta|0\rangle = \langle 0|\eta\rangle = 1$

$\langle 1|\eta\rangle = \langle\eta|1\rangle^* = -\eta$

$\langle\eta'|\eta\rangle = \exp(\eta'^*\eta) = 1 + \eta'^*\eta.$

$$\int d\eta^* d\eta\,\eta^*\eta = -1$$

$$\int d\eta^* d\eta\, e^{-\lambda\eta^*\eta} = \lambda$$

$$\int d\eta^* d\eta\, \exp(-\eta^*\eta)\,|\eta\rangle\langle\eta| = |0\rangle\langle 0| + |1\rangle\langle 1| = \mathbb{1}$$

$$\int d\eta^* d\eta\, \exp(-\eta^*\eta)\langle -\eta|A|\eta\rangle = \langle 0|\hat A|0\rangle + \langle 1|\hat A|0\rangle = \mathrm{Tr}\,\hat A$$

$$\langle\eta_i|e^{-\Delta\tau\hat H}|\eta_{i-1}\rangle = \langle\eta_i|1 - \Delta\tau\hat H|\eta_{i-1}\rangle = \langle\eta_i|\eta_{i-1}\rangle(1 - \Delta\tau H(\eta_i^*, \eta_{i-1}))$$

$$= \exp(\eta_i^*\eta_{i-1} - \Delta\tau H(\eta_i^*, \eta_{i-1})) \qquad // \exp(-\eta_i^*\eta_i)$$

$$Z = \mathrm{Tr}(e^{-\beta\hat H}) = \cancel{\int d\theta^* \int d\theta}\; e^{-\theta^*\theta} \int_{\eta(0)=\theta,\ \eta^*(\beta)=\theta^*}^{\eta^*(\beta) = -\theta^*} D\eta^* D\eta\; e^{-\int_0^\beta d\tau\,(\eta^*\partial_\tau\eta + H(\eta^*, \eta))}$$

$$= \int D\eta^* D\eta$$

$$\eta(\beta) = -\eta(0)$$

Теория поля:

$$\eta^*, \eta \to \bar\Psi, \Psi$$

$$Z = \int\limits_{\substack{\bar\Psi(0) = -\bar\Psi(\beta) \\ \Psi(0) = \Psi(\beta)}} D\bar\Psi D\Psi \, \exp\left(-\int_0^\beta d\tau \int d^3x \left(\bar\Psi \gamma^\mu_E \partial^E_\mu \Psi + m\bar\Psi\Psi\right)\right)$$

Евклидов лагранжиан Дирака.

$t \to -i\tau$
$iS \mapsto -S_{Eucl}$

$\{\gamma^\mu_E, \gamma^\nu_E\} = 2\delta_{\mu\nu}$

$$Z[J] = \dots$$

$$\dots \quad \mathcal{L}_E \to \mathcal{L}_E - \bar J_\Psi \Psi - \bar\Psi J_\Psi$$

$$\langle 0 | T(\bar\Psi(x_1)\Psi(x_2)) | 0 \rangle = \frac{1}{Z[0]} \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta \bar J(x_2)}$$

$$\int d\bar\eta \, d\eta \, \exp(-\bar\eta A \eta) = A$$
$$(1 - \bar\eta A \eta).$$

$$(\gamma^\mu \partial^E_{\mu(x)} + m)\Pi(x-y) = \delta^{(4)}(x-y)$$

$$\Pi(x-y) = \frac{1}{(2\pi)^3 \beta} \sum_n \int d^3p \, e^{i\omega_n(\tau_x - \tau_y) + ip^\mu(x^\mu - y^\mu)} \, \Pi(p)$$

$$(\gamma^0 i\omega_n + \gamma^i i p_i + m)\Pi(\vec p) = 1$$

$$\Pi(\vec p) = \frac{-i\gamma^0\omega_n - i\vec\gamma\vec p + m}{\omega_n^2 + p^2 + m^2}$$

$$\tau_x \to \tau_x + \beta$$

$$e^{i\omega_n\tau_x} \to e^{i\omega_n\tau_x} \cdot e^{i\omega_n\beta}$$

$e^{i\omega_n\beta} = -1$.

$i\omega_n\beta = i\pi(2n+1)$

$\omega_n = \frac{2\pi n + \pi}{\beta} = \frac{2\pi}{\beta}\left(n + \frac{1}{2}\right)$

частоты Matsubarovskije ~~частоты~~