Теорема Биркхофа: Решение Шварцшильда — единственное сферически/симметричное решение уравнений $\boxed{R_{ab}=0}$ (ур-й Эйнштейна в вакууме)

$\Updownarrow$

1) Нет сферически/симметричных гравитационных волн
2) Все сферически/симметричные решения ур-й Эйнштейна статичны!

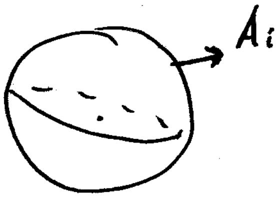
$\Updownarrow$

Ур-ния Максвелла: единственное сферически-симметричное решение — решение Кулона.

Задача

$$A_\mu : \quad \boxed{A_0 \,|\, A_i}$$

$$\boxed{\begin{aligned} A_0(r,t) &= \varphi(r,t) \\ A_i &= n_i\, a(r,t) \end{aligned}}$$



$$\partial_i A_i = \underbrace{\partial_i n_i}_{\frac{\delta_{ii} - 2n_i n_i}{r} = \frac{2}{r}} a(r,t) + \cancel{n_i}\, \partial_r a\, \cancel{n_i} = \frac{2}{r} a(r,t) + \partial_r a = 0$$

Калибровка

$$\partial_\mu F_{\mu\nu} = 0$$

$$\partial_\mu F_{\mu 0} = \cancel{\partial_0 F_{00}} + \partial_i F_{i0} = 0$$

$$\partial_i(\partial_i A_0 - \partial_0 a \ldots$$

$$\boxed{\partial_i a_i = 0} \Rightarrow \boxed{\Delta A_0 = 0} \Leftrightarrow \boxed{A_0 = \varphi = \frac{q(t)}{r}}$$

$$\partial_\mu F_{\mu i} = 0 = \underbrace{\partial_0 F_{0i}}_{\parallel} - \underbrace{\partial_j F_{ji}} = 0$$

$$\partial_0(\partial_0 a_i - \partial_i a_0)$$

$$- \partial_j(\partial_j a_i - \cancel{\partial_i a_j})$$

$$\boxed{\partial_0 \partial_0 a_i - \partial_j \partial_j a_i - \partial_i \partial_0 a_0 = 0}.$$

$$\frac{\partial_r a}{a} = -\frac{2}{r} \Leftrightarrow \partial_r \ln a = -2/r$$

$$\ln a = -2 \ln r + \ln a_0$$

$$\partial_i n_i = \partial_i \left(\frac{x_i}{x}\right) =$$

$$= \frac{3}{x} - \frac{x_i n_i}{x^2} = \frac{2}{x}$$

$$\boxed{a = \frac{a_0(t)}{r^2}}$$

$$\partial_i a_i = \partial_i \left(\frac{n_i}{r^2} a_0(t)\right) = \frac{2}{r^3} a_0 - \frac{2 \cancel{n_i n_i}}{r^3} a_0 = 0$$

$$\propto \frac{\# a_0 \, n_i}{r^4} \qquad \Leftrightarrow \boxed{a_0 = 0}$$

$$\frac{\ddot{a}_0 n_i}{r^2} - \Delta a_i + \frac{\dot{q} n_i}{r^2} = 0 \qquad \boxed{\dot{q} = 0} \qquad \text{ok}$$

$$\Delta a_i = \Delta(n_i a(r,t)) = \Delta\left(\frac{n_i a_0}{r^2}\right) =$$

$$= \partial_j \partial_j \left(\frac{n_i a_0}{r^2}\right) = a_0 \partial_j \left[\frac{\delta_{ij} - 3 n_i n_j}{r^3} - \cancel{\frac{2 n_i n_j}{r^3}}\right]$$

$$= a_0 \left(-\frac{3}{r^4}\right) n_i - 3 a_0 \underbrace{\partial_j \left(\frac{n_i n_j}{r^3}\right)}$$

$$+ a_0 \frac{3 n_j n_i n_j}{r^4} - \frac{3 a_0}{r^4} (\delta_{ij} - n_i n_j) n_j$$

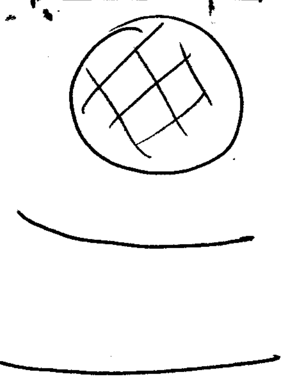
$$- 3 a_0$$

Redshift

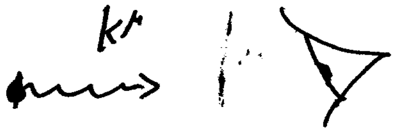
$$\boxed{ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2M}{r}} - r^2 d\Omega_2^2}$$

Импульс

Частота света



k^μ



Локально лоренцева СО

$$g^{\mu\nu}(x^0) \longrightarrow \eta^{\mu\nu}(x_0)$$

Стоящий наблюдатель

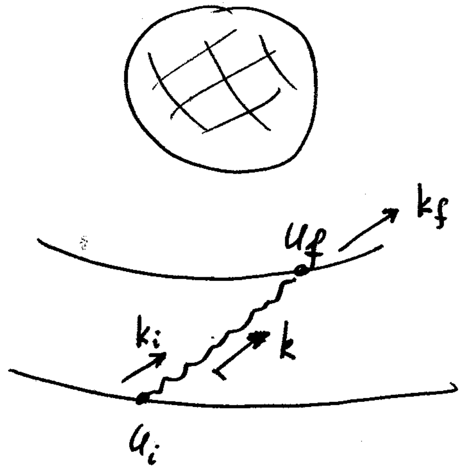
$$U_\mu^\mu = (1, 0, 0, 0)$$

$$\boxed{\omega = k\, U^\mu_\mu}$$

Не-стоящий наблюдатель: делаем Лоренцев буст.

$\Downarrow$

$$\boxed{\omega = k\, u_\mu} \leftarrow \text{Можно вычислять в любой СО}$$



$$\begin{cases} \omega_i = k_i U_i \\ \omega_f = k_f U_f \end{cases}$$

$$\boxed{\xi^\mu = \frac{\partial}{\partial t}}$$

$$\boxed{p_\mu = m\, U^\mu}$$

$$U^\mu = \frac{dx^\mu}{ds}$$

$$\begin{cases} U_i^\mu = \dfrac{\xi^\mu(x_i)}{\sqrt{\xi^2(x_i)}} \\ U_f = \dfrac{\xi^\mu(x_f)}{\sqrt{\xi^2(x_f)}} \end{cases}$$

$$\boxed{\omega_i = \frac{k_i \xi_i}{\sqrt{\xi_i^2}}} \qquad \boxed{\omega_f = \frac{k_f \xi_f}{\sqrt{\xi_f^2}}}$$

$$\boxed{(k_i \xi_i) = (k_f \xi_f)}$$

$$\boxed{\begin{matrix} k = const \\ m \to 0 \end{matrix}}$$

U^μ – Скорость

mU^μ – Импульс $\Big] \Rightarrow \boxed{k_\mu \propto U^\mu} \leftarrow \Big\{$ Импульс определяется как 4-скорость $\Big\}$

$\Downarrow$

$$\boxed{\omega_i \sqrt{\xi_i^2} = \omega_f \sqrt{\xi_f^2}}$$

$$\sqrt{\xi_i^2} = \sqrt{\xi_i^\mu g_{\mu\nu} \xi_i^\nu} = \sqrt{g_{00}} = \sqrt{1 - \frac{2M}{r_i}}$$

$$\boxed{\frac{\omega_i}{\omega_f} = \frac{\sqrt{1 - \frac{2M}{r_f}}}{\sqrt{1 - \frac{2M}{r_i}}}}$$

$$\boxed{\frac{\omega_f}{\omega_i} = \left(\frac{1 - 2M/r_i}{1 - 2M/r_f} \right)^{1/2}}$$

$\boxed{r_f \to \infty}$ – Наблюдатель находится на ∞ расстоянии

$$\boxed{\frac{\omega_f}{\omega_i} = \left(1 - 2M/r_i\right)^{1/2}}$$

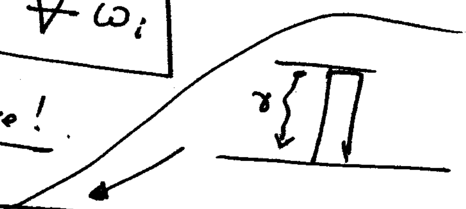
Red shift

Красное смещение

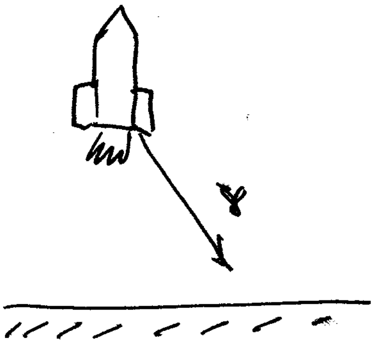
физически: $\boxed{E = \hbar\omega}$ $\boxed{ЗСЭ}$

$\boxed{r_i \to 2M} \Rightarrow \boxed{\omega_f \to 0 \quad \forall \omega_i}$

Бесконечное красное смещение!



Экспериментальная проверка: Pound & Rebka (1960) $\boxed{\delta = 1\%}$



$\delta = 0{,}01\,\%$

Vessot & Levine, 1979

~~Static Star~~

Статическая звезда: $\boxed{\dfrac{M}{R} \le 4/9}$

Максимальное красное смещение от звёзд

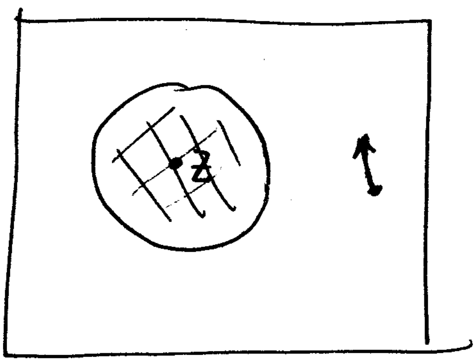
$$\boxed{\frac{\omega_f}{\omega_i} = \left(1 - \frac{2\cdot 4}{9}\right)^{1/2} = 1/3} \Rightarrow \boxed{Z_{max} = \frac{\omega_f}{\omega_i} - 1 = 2}$$

Квазары производят красное смещение > 2

$\Downarrow$

Не м. быть из-за гравитационных эффектов!

Геодезические

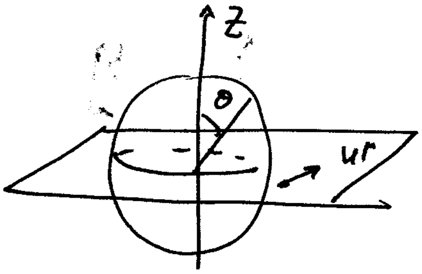


Симметрия относительно отражений

$$\boxed{z \mapsto -z}$$

$\Downarrow$

[Траектория не может выйти из плоскости $\theta = \frac{\pi}{2}$



$$\boxed{u^\mu = \frac{dx^\mu}{d\tau} = \dot{x}^\mu}$$

Времениподобные геодезические: τ - собственное время.

Светоподобные: τ - афинный параметр.

$$\varkappa = u^2 = g_{00}\left(\frac{dt}{d\tau}\right)^2 + g_{rr}\left(\frac{dr}{d\tau}\right)^2 + g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 =$$

$$= \left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{1}{1-\frac{2M}{r}}\dot{r}^2 - r^2\dot{\varphi}^2$$

$$\boxed{E = \xi u = \dot{t}\left(1 - \frac{2M}{r}\right)}$$

$$\boxed{\varkappa = \begin{cases} 0, & \text{Светоподобные} \\ 1, & \text{Простр.-подобные} \end{cases}}$$

$\tilde{\xi} \leftarrow$ Поле вращений φ

$$\boxed{\tilde{\xi} = \frac{\partial}{\partial\varphi}}$$

$$\boxed{L = -(\tilde{\xi}, u^\mu) = \dot{\varphi}(+r^2)}$$

– Момент количества движения

(и у фотона, и у массивной т-чки –

Закон Кеплера.

$$\boxed{\dot{\varphi} = \frac{L}{r^2}}$$

$$\boxed{\dot{t} = \frac{E}{1 - \frac{2M}{r}}}$$

$$\Rightarrow \boxed{\varkappa = \left(1 - \frac{2M}{r}\right)^{-1}E^2 - \frac{\dot{r}^2}{1-\frac{2M}{r}} - \frac{L^2}{r^2}}$$

$$V(r) = \left(\varkappa + \frac{L^2}{r^2}\right)\left(1 - 2M/r\right)\ \frac{1}{2}$$

Массивное движение

$\boxed{\varkappa = 1}$

$$\frac{dV}{dr} = \frac{d}{dr}\left(1 - \frac{2M}{r} + \frac{L^2}{r^2} - \frac{2ML^2}{r^3}\right) =$$

$$= \frac{2M}{r^2} - \frac{2L^2}{r^3} + \frac{6ML^2}{r^4} = 0$$

$$2M r^2 - 2L^2 r + 6ML^2 = 0$$

$$r_\pm = \frac{\cancel{2}1}{\cancel{4}2M}\left[+\cancel{2}L^2 \pm \cancel{2}\sqrt{\cancel{4}L^4 - \cancel{48}\,12\,M^2 L^2}\right]$$

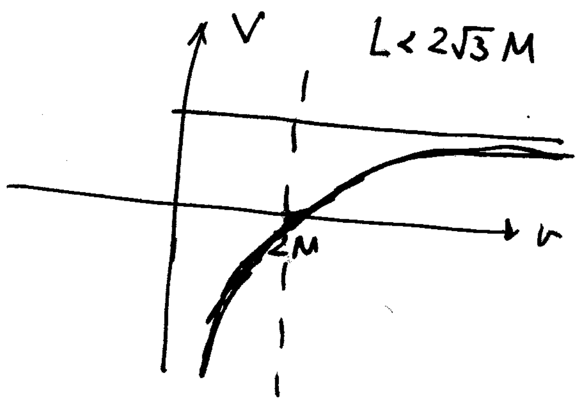
$$\boxed{r_\pm = \frac{L}{2M}\left[L \pm \sqrt{L^2 - 12M^2}\right]}$$

$\boxed{L > \sqrt{12}M}$

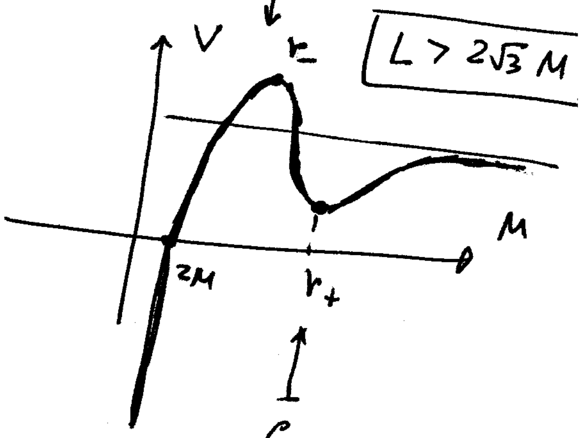
$\Downarrow$

Есть 2 экстремума

$\boxed{L < \sqrt{12}M}$ ($\sqrt{12} = 2\sqrt{3}$) $\Rightarrow$ Нет экстремумов



Неустойчивая орбита



Стабильная орбита

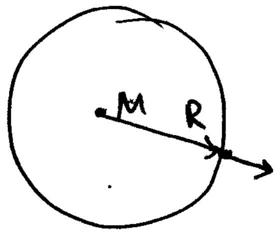
Круговая

$\boxed{L \gg M}$ $\quad$ $\boxed{R_+ \sim \frac{L^2}{M}}$ — Ньютоновский предел

$\Downarrow$

M — действительно масса!

Задача



$$\boxed{a = \frac{V^2}{R}}$$

$$ma = F$$

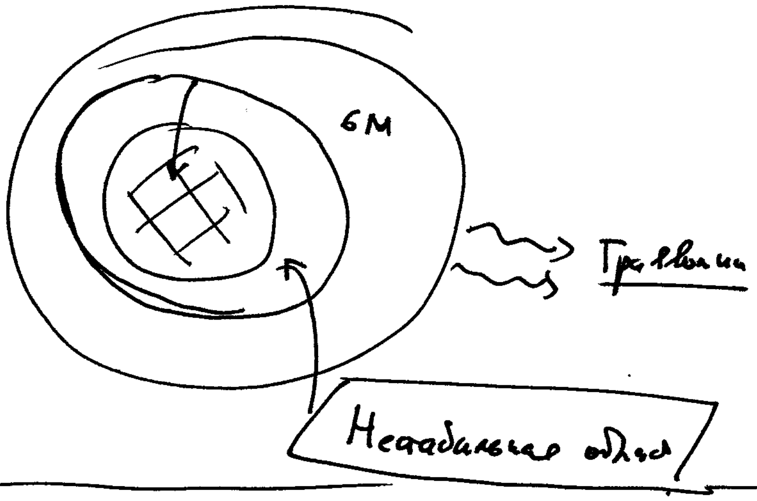
$$\cancel{m}\frac{V^2}{\cancel{R}} = \frac{\cancel{m}M}{R^{\cancel{2}}}$$

$$VR = L \Rightarrow \boxed{V = \frac{L}{R}} \Rightarrow \frac{L^2}{R^{\cancel{2}}} = \frac{M}{\cancel{R}} \Rightarrow \boxed{R = L^2/M}$$

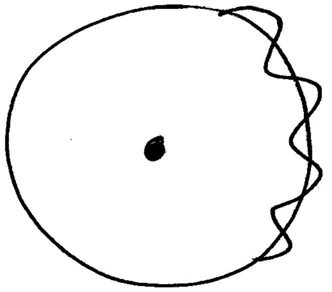
$$R_{+\,min}: \quad \boxed{L^2 = 12M^2} \qquad R_{+\,min} = \frac{2\sqrt{3}M \cdot 2\sqrt{3}M}{2M} = 6M$$

$\boxed{R_+ > 6M}$ ⟵ Стабильные орбиты не приближаются ближе, чем $\boxed{3\,R_s}$

Излучение грав-волн



Осцилляции на орбите



$$\omega_{eff}^2 = \left.\frac{d^2V}{dr^2}\right|_{R_+} =$$

$$= \frac{1}{2}\left(-\frac{4M}{r^3} + \frac{6L^2}{r^4} - \frac{24ML^2}{r^5}\right) =$$

$$R_+ = \frac{L}{2M}\left(L + \sqrt{L^2 - 12M^2}\right).$$

$$R_+^2 = \frac{1}{2M}\left(2L^2R_+ - 6ML^2\right)$$

$$\omega_{eff}^2 = \frac{1}{2R_+^5}\Big[+ \frac{\cancel{4M}}{\cancel{2M}}\big(\overset{4}{\cancel{2L^2}}R_+ - \overset{-12}{\cancel{6}}ML^2\big) + 2\cancel{6}L^2R_+ \cancel{-24ML^2}\Big] =$$

$$= \frac{1}{2R_+^5}\left[2L^2R_+ - 12ML^2\right] =$$

$$= \frac{L^2}{R_+^5}\left[R_+ - 6M\right]$$

$$= \frac{M(R_+ - 6M)}{R_+^3(R_+ - 3M)}$$

$$\boxed{r = R_+ + \delta r \cdot e^{i\omega t}}$$

$$+\frac{\omega^2\delta r^2}{2} = +V''\frac{\delta r^2}{2}$$

$$\Downarrow$$

$$\boxed{\omega^2 = V''(R_+)}$$

$$R_+^2 = \frac{L^2}{M}\left(R_+ - 3M\right)$$

$$\boxed{\frac{L^2}{R_+^2} = \frac{M}{R_+ - 3M}}$$

$$\boxed{\omega_r^2 = \frac{M(R_+ - 6M)}{R_+^3(R_+ - 3M)}}$$

Собственное время

$$\boxed{\delta r \propto \cos(\omega \tau)}$$

$$\omega_\phi = \frac{d\phi}{d\tau} = \frac{L}{R_+^2} = \frac{1}{R_+}\frac{\sqrt{M}}{\sqrt{R_+ - 3M}}$$

$$\boxed{\omega_\phi^2 = \frac{M}{R_+^2(R_+ - 3M)}}$$

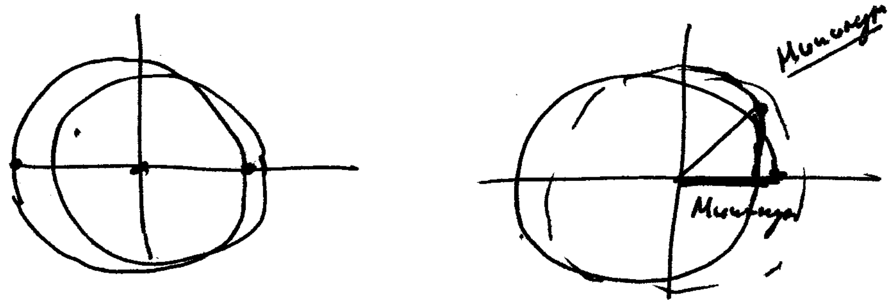
Частота движения по орбите по отношению к собственному времени.

Ньютон: $\boxed{R_+ \gg M}$

$$\boxed{\omega_r^2 \simeq \frac{M}{R_+^2} = \omega_\phi^2}$$

$$\Downarrow$$

Движение по эллипсу



$\boxed{\omega_r \neq \omega_\phi}$ $\Rightarrow$ ~~Сложение двух колебаний~~

Сложение двух гармонических движений

Орбиты не замыкаются!

$$\omega_p = \underbrace{\omega_r - \omega_\phi} =$$

Параметр:

$\begin{cases} T_{\varphi r} - \text{Мч достигл максимума} \\ T_{\varphi} - \text{Мч пришло в то же } \varphi \end{cases}$

$$\Delta\varphi = (T_r - T_\varphi)\,\omega_\varphi =$$

Смещение перигелия

$$\frac{\Delta\varphi}{T_\varphi} = \omega_p = \frac{T_r - T_\varphi}{T_\varphi}\,\omega_\varphi = \left(\frac{\frac{\cancel{2\pi}}{\omega_r} - \frac{\cancel{2\pi}}{\omega_\varphi}}{\cancel{2\pi}}\right)\omega_\varphi^2 =$$

$$\underbrace{= (\omega_\varphi - \omega_r)}$$

⟵ Частота смещения перигелия /// Юпитера.

Частота прецессии

$$\boxed{\omega_p = \omega_\varphi - \omega_r}$$

$$\omega_\varphi = \frac{\sqrt{M}}{R_+\sqrt{R_+ - 3M}}$$

$$\omega_r = \frac{\sqrt{M}\,\sqrt{R_+ - 6M}}{R_+^{3/2}\sqrt{R_+ - 3M}} = \frac{\sqrt{R_+ - 6M}}{\sqrt{R_+}}\,\omega_\varphi$$

$$\omega_p = -\omega_\varphi\left(\frac{\sqrt{R_+ - 6M}}{\sqrt{R_+}} - 1\right)$$

$$\omega_p = \omega_\varphi\left(\cancel{1} + \frac{3M}{R_+} - \cancel{1}\right) = \frac{3M\omega_\varphi}{R_+}$$

$$\boxed{\omega_p = \frac{3M^{3/2}}{R_+^{5/2}}}$$

$$\boxed{\omega_p = \frac{3\,G^{3/2}\,M^{3/2}}{R_+^{5/2}}}$$

$\text{кг}^{3/2}$ $= \frac{1}{\text{см}}$.

$\text{см}^{5/2}$

см^3

Можно упереть и посчитать масса!

$[G] = \frac{1}{\text{кг}^2}$

$$\boxed{\omega_p = 43\ \frac{\text{arcsec}}{100\ \text{yr}}}$$

$$\begin{cases} M_\odot = 2\cdot 10^{30}\ \text{кг} \\ R_+ = 6\cdot 10^{10}\ \text{м}. \end{cases}$$

~~[illegible]~~ меркурий

$43\ \frac{1}{60}\ \frac{2\pi}{360}\ \frac{1}{100\ \text{лет}}\ \frac{1}{60} = \omega$

$$\boxed{T_p = 3.5\cdot 10^6\ \text{лет}}$$

$$T = \frac{2\pi}{\omega} = 1\ \text{год} \times 3\cdot 10^6 \quad 6\text{b}$$

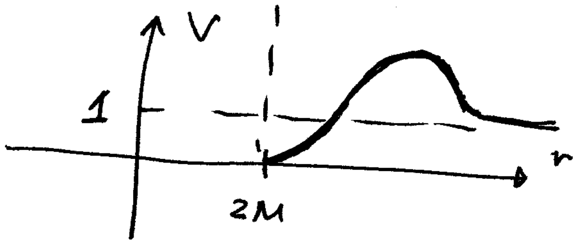
$$-\frac{1}{2}\dot{r}^2 + \frac{1}{2}E^2 - \frac{1}{2}\varkappa\left(1-\frac{2M}{r}\right) - \frac{1}{2}\frac{L^2}{r^2}\left(1-\frac{2M}{r}\right) = 0.$$

$$\boxed{\frac{1}{2}\dot{r}^2 + V(r) = \frac{1}{2}E^2}$$

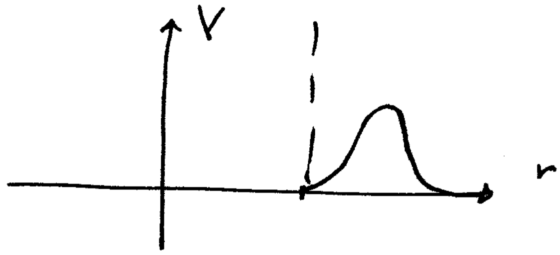
Барјер ф-ја круго

$$\boxed{V(r) = \frac{1}{2}\left(1-\frac{2M}{r}\right)\left(\varkappa + \frac{L^2}{r^2}\right)}$$

Масивни



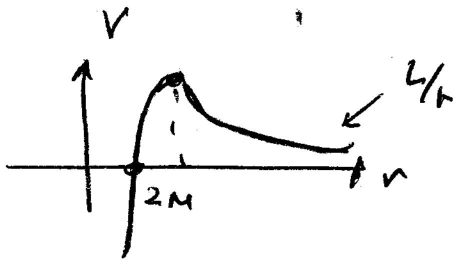
Безмасени



$$\cancel{0 = \frac{dV}{dr} = \frac{d}{dr}\left(\frac{L^2}{r^2} - \frac{2ML^2}{r^3}\right) = -2\frac{L^2}{r^3} + 6\frac{ML^2}{r^4} = 0}$$

Светоподобные геодезические

$$\boxed{V(r) = \frac{L^2}{2r^3}(r-2M)}$$



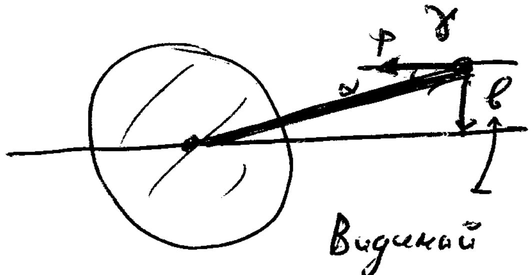
Максимум: $$-\frac{3\cancel{L^2}}{\cancel{2r^4}}(r-2M) + \frac{\cancel{L^2}\, r}{\cancel{2r^3}} = 0$$

$$-2r + 6M = 0 \quad \Rightarrow \quad \boxed{r = 3M}$$

Для модели

$$\frac{1}{2}E^2_{min} = \frac{L^2 \cancel{M}}{2(3\cancel{M})^3}\,\frac{(\cancel{3M-2M})}{M^2} = \frac{L^2}{2\, M^2\, 27}$$

$$\boxed{\frac{L^2}{E^2_{min}} = 27M^2}$$



Видимый прицельный параметр

$\boxed{r \gg M} \Rightarrow \boxed{\text{Плоское пространство}}$

$\Downarrow$

$$L = [\vec{p} \times \vec{r}]_z = p \cdot r \sin\alpha = pb$$

$\boxed{\frac{L}{E} = b}$ — Прицельный параметр

$\boxed{b_c < 3\sqrt{3}M}$ ← Сечение захвата фотонов

$$\boxed{\sigma = \pi b_c^2 = 27\pi M^2}$$

$$\dot\phi = \frac{L}{r^2}$$

$$\dot r = \pm\sqrt{-2V(r) + E^2}$$

$$\frac{d\phi}{dr} = -\frac{L}{r^2}\,\frac{1}{\sqrt{\dots}}$$

$$\boxed{\frac{d\phi}{dr} = -\frac{L}{r^2}\left[E^2 - \frac{L^2}{r^3}(r-2M)\right]^{-1/2}}$$

$$\boxed{\Delta\phi = \phi(+\infty) - \phi(-\infty)}$$

$$\boxed{b = \frac{L}{E}}$$

$$\frac{d\phi}{dr} = \pm\frac{b}{r^2}\left[1 - \frac{b^2}{r^3}(r-2M)\right]^{-1/2}$$

$$\frac{d\phi}{dr} = 0 \Rightarrow \boxed{\text{Turning point}} \qquad \boxed{b^2(r_0 - 2M) = r_0^3}$$

$$\frac{\Delta\phi}{2} = \int_{r_0}^{\infty} dr \, \frac{b}{\left[r^4 - b^2 r(r-2M)\right]^{1/2}} = \int_{1/r_0}^{0} \frac{du \; b}{\left[1 - b^2u^2(1-2Mu)\right]^{1/2}}$$

$$\boxed{u = 1/r}$$

$$\boxed{M = 0}$$

$$\Downarrow$$

$$\frac{\Delta\phi}{2} = \int_{1/r_0}^{0} \frac{du \cdot b}{\left[1 - b^2u^2\right]^{1/2}} = \arcsin(1) - \arcsin(0)$$

$$= \pi/2 \quad \text{ok.}$$

Разложимся по M/r

$$\boxed{b^2 (r_0 - 2M) = r_0^3}$$

$$\boxed{b^2 = \left(\frac{r_0^3}{r_0 - 2M} \right)}$$

$$\frac{\Delta\phi}{2} = \int_0^{1/r_0} \frac{du \; r_0^{3/2} / \cancel{\sqrt{r_0 - 2M}}}{\left[1 - \frac{r_0^3 \, u^2}{\cancel{r_0 - 2M}} (1 - 2Mu) \right]^{1/2}} =$$

$r_0 - 2M$

$$= \int_0^{1/r_0} \frac{du}{\left[\frac{1}{r_0^2} - \frac{2M}{r_0^3} - u^2 + 2Mu^3 \right]^{1/2}}$$

$$\frac{\partial \Delta\phi}{2 \partial M} = -\cancel{\tfrac{1}{2}} \int_0^{1/r_0} \frac{du \left(-\frac{\cancel{2}}{r_0^3} + \cancel{2} u^3 \right)}{\left[1/r_0^2 - \cancel{\frac{2M}{r_0^3}} - u^2 + \cancel{2Mu^3} \right]^{3/2}}$$

$$= - \int_0^{1/r_0} \frac{du \left(u^3 - 1/r_0^3 \right)}{\left[1/r_0^2 - u^2 \right]^{3/2}}$$

$$= -\frac{1}{r_0} \int_0^1 \frac{du' \cancel{1/r_0} \, \cancel{1/r_0^3} \, (u'^3 - 1)}{\cancel{1/r_0^3} \, (1 - u'^2)^{3/2}}$$

$$= + 1/r_0 \int \frac{du' \left(u' (-u'^2 + 1) - u' + 1 \right)}{(1 - u'^2)^{3/2}}$$

$$= 1/r_0 \left[\int_0^1 \frac{du' \, u'}{\ldots} - \int^1 \frac{u' du'}{\ldots} + \int^1 \frac{du'}{(\ldots)^{3/2}} \right.$$

$$I(\ell) = \int \frac{du'}{\sqrt{\ell - u'^2}} = \cancel{\ldots} \; \frac{1}{\sqrt{\ell}} \int \frac{du'/\sqrt{\ell}}{\sqrt{1 - u'^2/\ell}}$$

$$= \arcsin\left(\frac{u'}{\sqrt{\ell}}\right)$$

$$\underbrace{\frac{\partial I}{\partial \ell} = -\frac{1}{2} \int \frac{du'}{\left(\sqrt{\ell - u'^2}\right)^3}}$$

$$\Downarrow$$

$$\int \frac{du'}{\left(\sqrt{1 - u'^2}\right)^3} = -2 \left.\frac{\partial I}{\partial \ell}\right|_{\ell=1} = (+2)\, \frac{1}{\sqrt{1 - \frac{u'^2}{\ell^2}}} \left(+\frac{u'}{\ell\, \ell^{3/2}}\right)\Bigg|_{\ell=1} =$$

$$= \frac{u'}{\sqrt{1 - u'^2}}$$

$$\left.\frac{\partial \Delta\phi}{2\partial M}\right|_{M=0} = 1/r_0 \left[\; - \underbrace{\sqrt{1 - u'^2}}\Big|_0^1 - \frac{1}{\sqrt{1 - u'^2}}\Big|_0^1 + \right.$$

$$\left. + \frac{u'}{\sqrt{1 - u'^2}}\Big|_0^1 \right] =$$

$$= 1/r_0 \left[\cancel{+\frac{\sqrt{-1 + u'}}{\sqrt{1 - u'^2}}} + 1 + 1 \right] = 2/r_0$$

$$\boxed{\left.\frac{\partial \Delta\phi}{\partial M}\right|_{r_0 = \text{const}} = 4/r_0}$$

$$\boxed{\Delta\phi = \pi + 4/b\, M}$$

$$\boxed{\delta\phi = \frac{4M}{b}} = \boxed{\frac{4MG}{bc^2}} = \boxed{1{,}75''}$$

Дузон, Eddington, Davidson, 1920 ... шой

<u>Видимый свет</u>

[Солне... ...ение +
Эксцед...

Radio waves 1% Fomalont, Sramek, ...