Фундаментальные представления современной физики:

от взаимодействий элементарных частиц до структуры и эволюции Вселенной

Лекция 3


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$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A + \lambda B =$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} + \begin{pmatrix} \lambda b_{11} & \lambda b_{12} & \lambda b_{13} \\ \lambda b_{21} & \lambda b_{22} & \lambda b_{23} \\ \lambda b_{31} & \lambda b_{32} & \lambda b_{33} \end{pmatrix} =$$

$$\begin{pmatrix} a_{11} + \lambda b_{11} & a_{12} + \lambda b_{12} & a_{13} + \lambda b_{13} \\ a_{21} + \lambda b_{21} & a_{22} + \lambda b_{22} & a_{23} + \lambda b_{23} \\ a_{31} + \lambda b_{31} & a_{32} + \lambda b_{32} & a_{33} + \lambda b_{33} \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} =$$

$$\begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{32} & a_{11} \cdot b_{13} + a_{12} \cdot b_{23} + a_{13} \cdot b_{33} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + a_{23} \cdot b_{32} & a_{21} \cdot b_{13} + a_{22} \cdot b_{23} + a_{23} \cdot b_{33} \\ a_{31} \cdot b_{11} + a_{32} \cdot b_{21} + a_{33} \cdot b_{31} & a_{31} \cdot b_{12} + a_{32} \cdot b_{22} + a_{33} \cdot b_{32} & a_{31} \cdot b_{13} + a_{32} \cdot b_{23} + a_{33} \cdot b_{33} \end{pmatrix}$$

$$(AB)_{\cdot 1} =$$

$$a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$
$$a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31}$$
$$a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31}$$

$$(AB)_{.2} =$$

$$\begin{matrix} a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \\ a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} \end{matrix}$$

$$(AB)_{\cdot 3} =$$

$$\begin{matrix} a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{matrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AI = IA = A$$

$$TrA = a_{11} + a_{22} + a_{33}$$

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

$$q(t) = q_0 \sin(\omega t + \phi)$$

$$H(p, q) = \frac{p^2}{2M} + \frac{M\omega^2}{2} q^2, \qquad M = 1.$$

$$H = \frac{P^2}{2} + \frac{\omega^2}{2} Q^2$$

$$[Q, P] = i\hbar I$$

$$\Psi = \psi(x), \quad Q\psi(x) = x\psi(x)$$

$$P\psi(x) = -i\hbar \frac{\partial}{\partial x} \psi(x)$$

$$a^- = \frac{1}{\sqrt{2\hbar\omega}}(\omega Q + iP)$$

$$a^+ = \frac{1}{\sqrt{2\hbar\omega}}(\omega Q - iP)$$

$$[a^-, a^+] = 1$$

$$H = \frac{\hbar\omega}{2}\left(a^{-}a^{+} + a^{+}a^{-}\right) =$$

$$\hbar\omega a^{+}a^{-} + \frac{\hbar\omega}{2}$$

$$[a^-, H] = \hbar\omega a^-$$
$$[a^+, H] = -\hbar\omega a^+$$

$$a^- : E \Rightarrow E - \hbar\omega$$
$$a^+ : E \Rightarrow E + \hbar\omega$$

$$H\psi_n = E_n\psi_n$$

$$a^-\psi_0 = 0$$

$$\psi_0, \quad \psi_1 \sim a^+\psi_0, \ldots,$$
$$\psi_n \sim (a^+)^n \psi_0, \ldots$$

$$E_0 = \frac{\hbar\omega}{2}, E_1 = \hbar\omega + \frac{\hbar\omega}{2}, \ldots,$$
$$E_n = n\hbar\omega + \frac{\hbar\omega}{2}, \ldots$$

$$\psi_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \psi_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \psi_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \cdots$$

$$\varphi = C_0\psi_0 + C_1\psi_1 + \ldots = \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ \vdots \end{pmatrix}$$

$$
a^{-} = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \dots \\ 0 & 0 & \sqrt{2} & 0 & \dots \\ 0 & 0 & 0 & \sqrt{3} & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}
$$

$$a^+ = \begin{pmatrix} 0 & 0 & 0 & 0 & \ldots \\ \sqrt{1} & 0 & 0 & 0 & \ldots \\ 0 & \sqrt{2} & 0 & 0 & \ldots \\ 0 & 0 & \sqrt{3} & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

$$a^+ a^- = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & \dots \\ 0 & 0 & 0 & 3 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$H = \hbar\omega a^{+}a^{-} + \frac{\hbar\omega}{2} =$$

$$\begin{pmatrix} \frac{\hbar\omega}{2} & 0 & 0 & 0 & \dots \\ 0 & \frac{3}{2}\hbar\omega & 0 & 0 & \dots \\ 0 & 0 & \frac{5}{2}\hbar\omega & 0 & \dots \\ 0 & 0 & 0 & \frac{7}{2}\hbar\omega & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$\psi_0(x) = C \exp\left(-\frac{\omega x^2}{2\hbar}\right)$$

$$|\psi(x)|^2 \neq 0, \quad -\infty < x < +\infty$$

$$H_F = \frac{\hbar\omega}{2}\left(a^+a^- - a^-a^+\right)$$

$$[a^-, H_F] = \hbar\omega a^-$$
$$[a^+, H_F] = -\hbar\omega a^+$$

$$a^- : E \Rightarrow E - \hbar\omega$$
$$a^+ : E \Rightarrow E + \hbar\omega$$

$$\{a^-, a^+\} = a^- a^+ + a^+ a^- = 1$$

$$(a^-)^2 = (a^+)^2 = 0$$

$$H_F = \hbar\omega a^+ a^- - \frac{\hbar\omega}{2}$$

$$a^- \psi_0 = 0, \quad a^+ \psi_0 = \psi_1$$

$$a^+ \psi_1 = 0$$

$$\psi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a^- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad a^+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$