# Спонтанное нарушение симметрии

## Абелев пример

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu \phi|^2 - V(\phi)}$$

$$\boxed{V(\phi) = -\mu^2 |\phi|^2 + \frac{\lambda}{2}|\phi|^4}$$

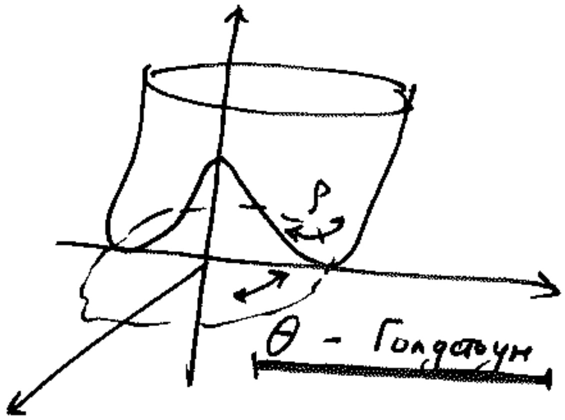
$$D_\mu \phi = \partial_\mu \Phi + ieA_\mu \phi$$

$$A_\mu \mapsto A_\mu - \frac{1}{e}\partial_\mu \alpha$$

$$\Phi \mapsto \Phi e^{i\alpha}$$

$$\boxed{\Phi = \rho e^{i\theta}}$$

$$\left\{\begin{array}{l} \rho \mapsto \rho \\ \theta \mapsto \theta + \alpha \end{array}\right\}$$ Калибровочное преобразование




$$V(\phi) = V(\rho) = -\mu^2\rho^2 + \frac{\lambda}{2}\rho^4$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |\partial_\mu \rho \cancel{e^{i\theta}} + i\rho\partial_\mu\theta \cancel{e^{i\theta}} + ieA_\mu \rho \cancel{e^{i\theta}}|^2 + V(\rho)$$

$$= -\frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu\rho)^2 + (\partial_\mu\theta + eA_\mu)^2\rho^2 + V(\rho)$$

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Минимум: $\boxed{\rho = v}$ $\quad -\cancel{2}\mu^2\cancel{\rho} + \cancel{2}\lambda\rho^{\cancel{3}\,2} = 0$

$$\boxed{v = \mu/\sqrt{\lambda}}$$

$$\boxed{\rho = v + \frac{h}{\sqrt{2}}}$$

$$\boxed{m_A = \sqrt{2}\, e v}$$

$$\boxed{m_h = \sqrt{2\lambda}\, v}$$

$$V(\rho) = V(v) - \cancel{\mu^2}\frac{h^2}{2} + \frac{1}{2}\frac{\lambda}{2}\cdot 4\cdot 3\, v^2 \frac{h^2}{2} = V(v) + \lambda v^2 h^2$$

Унитарная калибровка : $\boxed{\theta = 0}$

Но: Удобнее использовать более широкий класс $R_\xi$ - калибровок

$$I = \int [dA_\mu][d\phi\, d\phi^*]\; \delta(\partial_i A_i)\, \det[-\Delta]\; e^{iS[A,\phi]}$$

Также квантуем в Кулоновской калибровке

$$\boxed{\chi = +G - \frac{1}{\sqrt{\xi}}\left(\partial_\mu A^\mu - \xi e v^2 2\theta\right)}$$

$$= 2\xi e v^2 \theta$$

$$\chi^{(\alpha)} = +G - \frac{1}{\sqrt{\xi}}\left(\partial_\mu A_\mu - \underbrace{\frac{1}{e}\Box\alpha}\, - \xi e\theta - \xi e \alpha v^2 2\right) =$$

$$= \chi + \frac{1}{\sqrt{\xi}\, e}\Box\alpha + 2\sqrt{\xi}\, e\alpha v^2$$

$$\Delta_G(A,\phi]\int_0^{2\pi} d\alpha\; \delta(\chi^\alpha) = 1$$

$$\Delta_G(A,\phi)\; \left|\det\left[\frac{\Box}{\sqrt{\xi}\, e} + \sqrt{\xi}\, e\, 2v^2\right]\right|^{-1} = 1$$

Инвариантна

$$\boxed{\Delta_G(A,\Phi) = \left|\det\left[\frac{\Box}{\sqrt{\xi}\, e} + \sqrt{\xi}\, e\, 2v^2\right]\right|}$$

$$M = \frac{1}{e\sqrt{\xi}}\left(\Box + \xi m_A^2\right)$$

$$I = \int [dA_\mu][d\phi\, d\phi^*] \; \Delta_c(A)\; \delta(\partial_i A_i)\, e^{iS[A,\varphi]} \times$$

$$\times \int [d\alpha]\; \underbrace{\Delta_G(A,\phi)}_{\det(M_G).}\; \delta(\chi_G)$$

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$$I = \int [dA_\mu]\,[d\phi\, d\phi^*] \left|\det\left(\frac{\Box}{\sqrt{\xi}\, e} + 2\sqrt{\xi}\, e v^2\right)\right| \delta\left(G - \frac{\partial_\mu A^\mu}{\sqrt{\xi}} + \sqrt{\xi}\, e\theta\right) v$$

$$\times\; e^{iS[A,\phi]}$$

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Не зависит от $G$

$$\Downarrow$$

$$I \propto \int e^{-\frac{i}{2}\int G^2 d^4x}\, dG\; I$$

$$\Downarrow$$

$$I = \int [dA_\mu]\,[d\phi\, d\phi^*]\; |\det M_G|\; e^{\underbrace{iS[A,\phi] - \frac{i}{2\xi}\int d^4x\, (\partial_\mu A_\mu - \xi e\theta v^2)^2}_{iS_\xi}}$$

$$\boxed{S_\xi = -\frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu \Phi)^2 - \frac{1}{2\xi}(\partial_\mu A_\mu - 2\xi e\theta)^2}$$

$$S^{(2)} = -\frac{1}{4}F_{\mu\nu}^2 + \left|\frac{\partial_\mu h}{\sqrt{2}} + ieA_\mu v - i\partial_\mu \theta v\right|^2 -$$

$$-\frac{1}{2\xi}(\partial_\mu A_\mu - 2\xi e\theta v^2)^2 =$$

$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{(\partial_\mu h)^2}{2} + e^2 v^2 A_\mu^2 + v^2(\partial_\mu \theta)^2 -$$

$$\cancel{-2ev^2 A_\mu \partial_\mu \theta} - \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \cancel{+2\frac{1}{2\xi}\cdot \xi e v^2 \theta \partial_\mu A_\mu} -$$

$$-2\xi e^2 v^4 \theta^2 - m_h^2 h^2/2$$

$$S^{(2)} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{(\partial_\mu h)^2}{2} + \frac{1}{2} m_A^2 A_\mu^2 + 2v^2\left[\frac{(\partial_\mu \theta)^2}{2} + \frac{2\xi e^2}{2} v^2 \theta^2\right] - \frac{1}{2\xi}(\partial_\mu A_\mu)^2$$

$$\boxed{m_\theta^2 = \xi m_A^2}$$

II

$A_\mu$ находится в $\alpha$-калибровке

$\theta$ имеет массу $\sqrt{\xi}\, m_A = m_\theta$

$h$ - Хиггс

$$h \text{ - - - - - } h = \frac{i}{p^2 - m_h^2}$$

$$\theta \text{ - - - - - } \theta = \frac{i}{p^2 - \xi^2 m_A^2}$$

$$-\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A_\mu)^2\left(1 - \frac{1}{\xi}\right) + \frac{1}{2} m_A^2 A_\mu^2$$

$$A_\mu \frac{1}{2} \underbrace{\left(+g_{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial_\mu \partial_\nu + m_A^2 g_{\mu\nu}\right)}_{L^{\mu\nu}} A_\nu$$

$$\boxed{L^{\mu\nu} D_{\nu\lambda} = g_{\mu\lambda}\, i\,\delta(x - x')}$$

$$\mu \overset{k}{\rightsquigarrow} \nu = \frac{-i}{k^2 - m_A^2 + i0}\left(g_{\mu\nu} - (1 - \xi)\frac{k^\mu k^\nu}{k^2 - \xi m_A^2 + i0}\right)$$

$$\left(-k^2 g_{\mu\nu} + k_\mu k_\nu \left(1 - 1/\xi\right) + m_A^2 g_{\mu\nu}\right) \frac{(-i)}{k^2 - m_A^2 + i0} \times$$

$$\times \left(g_{\nu\lambda} - (1-\xi)\frac{k_\nu k_\lambda}{k^2 - \xi m_A^2}\right) = i\, g_{\mu\lambda}$$

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$$\frac{-i}{k^2 - m_A^2}\left(-k^2 g_{\mu\lambda} + m_A^2\right) + \left(k_\mu k_\lambda \left(1 - 1/\xi\right)\right) + \cancel{m_A^2 g_{\mu\lambda}} -$$

$$- (-k^2 + m_A^2)(1-\xi)\frac{k_\mu k_\lambda}{k^2 - \xi m_A^2} + \frac{k_\mu k^2 k^\lambda \cancel{(\xi - 1/\xi)}}{\xi (k^2 - \xi m_A^2)}(1-\xi)^2$$

То - что нужно для правой части

$$\frac{k_\mu k_\lambda}{\xi (k^2 - \xi m_A^2)}\Big[\underbrace{(\xi - 1)(k^2 - \cancel{\xi m_A^2}) - (k^2 - \cancel{m_A^2})(\xi - 1)\xi}_{-k^2(\xi-1)^2}\Big]$$

Ok

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$\boxed{k^2 \mapsto -}$ $\Rightarrow$ Хорошее поведение пропагатора

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Гости

$$\boxed{\mathcal{L}_c = i c^+ \hat{M} c = \frac{-i}{\sqrt{\xi} e} c^+ (\Box + \xi e^2 v^2) c}$$

$\xi m_A^2$

! Заметим, что духи полностью отцеплены от всех полей.

Это - особенность теории $U(1)$!

Зависимость от $\xi$ также как и зависимость от $\alpha$ должна сократиться!

**Фермионы**

$$\boxed{\mathcal{L}_f = \bar\Psi_L\, i\gamma^\mu D_\mu \Psi_L + \bar\Psi_R\, i\gamma^\mu \partial_\mu \Psi_R - \lambda_f\, \bar\Psi_L \Phi \Psi_R - \lambda_f\, \bar\Psi_R \Phi^* \Psi_L}$$

Как в электрослабой теории левые фермионы – заряжены, правые – незаряжены

$$\boxed{\begin{aligned} \Psi_L &\mapsto e^{i\alpha}\Psi_L \\ \Psi_R &\mapsto \Psi_R \end{aligned}}$$

Как же написать массы? $\Rightarrow$ Юкавку

$\Phi = v \Rightarrow$

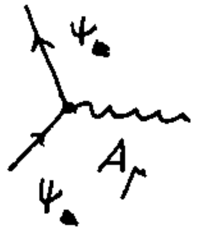
$$\mathcal{L}_f^{(2)} = \underbrace{\bar\Psi_L\, i\gamma^\mu \partial_\mu \Psi_L + \bar\Psi_R\, i\gamma^\mu \partial_\mu \Psi_R}_{\bar\Psi \gamma^\mu \partial_\mu \Psi} - \underbrace{\lambda_f v \bar\Psi_L \Psi_R - \lambda_f v \bar\Psi_R \Psi_L}_{\lambda_f \cdot v \cdot \bar\Psi \Psi}$$

$$\boxed{m_\psi = \lambda_f \cdot v}$$

Какие бывают вершины?

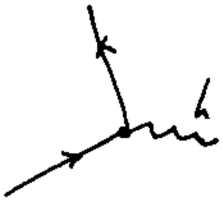
$$\bar\Psi_L\, i\gamma^\mu(\partial_\mu + ieA_\mu)\Psi_L \left(- \lambda_f v \bar\Psi_L i\theta \Psi_R - \right.$$

$- \lambda_f v \bar\Psi$ [illegible]




$+ie\,\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)$


$-\frac{m_\psi \gamma^5}{\sqrt{2}\,v}$




$+\frac{i m_\psi}{\sqrt{2}}$

$-\lambda_f\,\bar\psi_L \Phi^* \psi_R +$

$+\lambda_f\,\bar\psi_R \Phi^* \psi_L =$

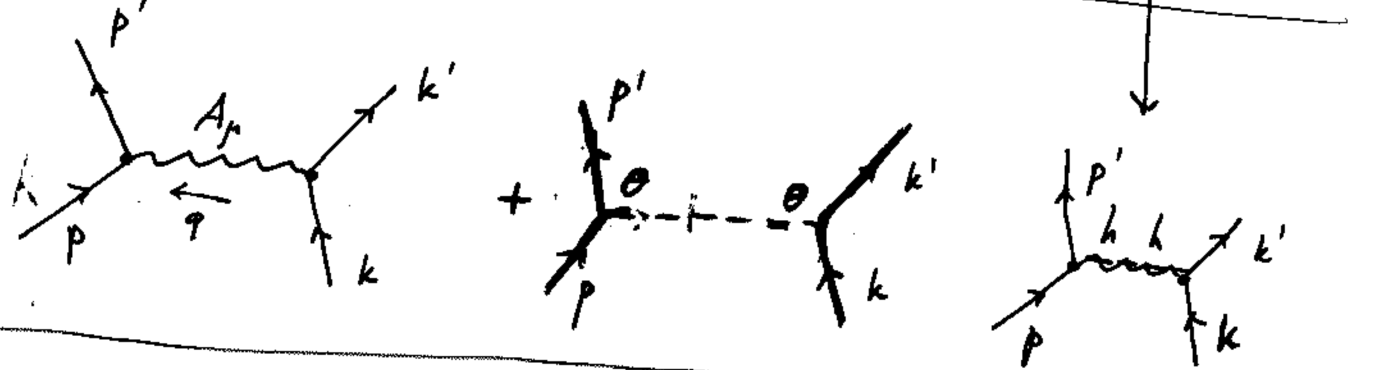
$= -\lambda_f\,\bar\psi\, i\theta v \left(\frac{1+\gamma^5}{2}\right)\psi$

$-\cancel{\lambda_f\,\bar\psi(+i\theta)v\,\frac{-1+\gamma^5}{2}\psi}$

$= -\frac{\lambda_f i v}{2}\,\bar\psi\gamma^5\psi$

$(\bar\psi\gamma^5\psi)^+ = -\bar\psi\gamma^5\psi$

$\boxed{\text{Independent of } \xi}$




$-e^2\,\bar u(p')\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u(p)$

$\bar u(k')\gamma^\nu\left(\frac{1+\gamma^5}{2}\right)u(k)\,\times$

$\times\;\frac{-i}{q^2-m_A^2}\left\{g_{\mu\nu}-(1-\xi)\frac{q_\mu q_\nu}{q^2-\xi m_A^2}\right\}$

$\frac{-i}{q^2-m_A^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_A^2}+\right.$

$\left.+\;q_\mu q_\nu\underbrace{\left(\frac{1}{m_A^2}+\frac{-1+\xi}{q^2-\xi m_A^2}\right)}\right)$

$\frac{q^2-\cancel{\xi m_A^2}-m_A^2+\cancel{\xi m_A^2}}{m_A^2(q^2-\xi m_A^2)}$

$\shortparallel$

$\bar u(p')\left(-\frac{m_\psi\gamma^5}{\sqrt{2}\,v}\right)u(p)\,\frac{i}{(p'-p)^2-\xi m_A^2}\,\times$

$\times\,\bar u(k')\left(-\frac{m_\psi\gamma^5}{\sqrt{2}\,v}\right)u(k)$

$h$

$\frac{\lambda_f^2}{2}\,\bar u(p')\gamma^5 u(p)\,\frac{i}{\underbrace{(p'-p)^2}_{q^2}-\xi m_A^2}\,\times$

$\times\,\bar u(k')\gamma^5 u(k)$

$$= \frac{-i}{q^2-m_A^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_A^2}\right) + \frac{q_\mu q_\nu}{m_A^2}\,\frac{(-i)}{q^2-m_A^2\xi}$$

II

$$= -e^2\,\bar u(p')\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u(p)\;\bar u(k')\gamma^\nu\left(\frac{1+\gamma^5}{2}\right)u(k)\times\left\{-\frac{i}{q^2-m_A^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_A^2}\right)\right\}.$$

$\boxed{\text{Не содержит } \xi}$

$$\begin{cases} -e^2\,\bar u(p')(\hat p'-\hat p)\left(\frac{1-\gamma^5}{2}\right)u(p) \\ \bar u(k')(\hat k'-\hat k)\left(\frac{1-\gamma^5}{2}\right)u(k)\,\frac{(-i)}{q^2-m_A^2\xi}\,\frac{1}{m_A^2}\end{cases}$$

$$\hat k\, u(k) = m_\psi\, u(k) = \lambda_f\cdot v\, u(k)$$

$$\bar u(k)\hat k' = -\lambda_f\cdot v\,\bar u(k)$$

$$\cancel{+e^2\,\frac{(+i)}{q^2-m_A^2\xi}\,\frac{1}{[illegible]}\,\frac{2(\lambda_f\cdot v)^2}{2e^2v^2}\,\bar u(p')\frac{1+\gamma^5}{2}u(p)\;\bar u(k')\frac{1+\gamma^5}{2}u(k)}$$

$$\cancel{m_A=\sqrt2\,e v}$$

$$q^\mu\,\bar u(p')\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u(p) =$$

$$= \cancel{(p'-p)}\,\frac{1}{2}\,\bar u(p')\left[\hat p'-\hat p-(\hat p'-\hat p)\gamma^5\right]u(p)$$

$$= \frac{1}{2}\, \bar{u}(p')\left[\hat{p}'\cdot\left(\frac{1-\gamma^5}{\text{[redacted]}}\right) - \left(\frac{1+\gamma^5}{\text{[redacted]}}\right)\hat{p}\right] u(p) =$$

$$= \frac{1}{2}\, \bar{u}(p')\left[+ m_\psi (1-\gamma^5) - m_\psi (1+\gamma_5)\right] u(p)$$

$$\begin{cases} \hat{p}\psi = m_\psi \psi. & \gamma^{0+} = \gamma^0 \quad \Big| \quad \hat{p}^+\gamma^0 = \gamma^0 \hat{p} \\ & \gamma^{i+} = -\gamma^i \\ \psi^+ \hat{p}^+ = m_\psi \psi^+ & \\ \psi^+\gamma^0\hat{p} = m_\psi \psi^+\gamma^0 \Rightarrow \boxed{\bar{\psi}\hat{p} = m_\psi \bar{\psi}} & \end{cases}$$

$$= -\bar{u}(p')\, m_\psi \gamma^5 u(p)$$

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 $= + e^2 (m_\psi)^2 \quad \bar{u}(p')\gamma^5 u(p)\, \bar{u}(k')\gamma^5 u(k) \times$

$\lambda_\psi^2 v^2 \leftarrow \qquad \times -\frac{1}{m_A^2} \quad \frac{+i}{q^2 - m_A^2 \xi}$

$$= -\frac{e}{2}\, \frac{i\lambda_\psi^2}{q^2 - m_A^2\xi}\, \bar{u}(p')\gamma^5 u(p)\, \bar{u}(k')\gamma^5 u(k)$$

$\Rightarrow$ Ok Сопряжение

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$$q^\mu\, \bar{u}(k')\gamma^\mu \frac{1-\gamma^5}{2} u(k) = (k-k')^\mu \bar{u}(k')\gamma^\mu \frac{1-\gamma^5}{2} u(k) =$$

$$= \frac{1}{2}\, \bar{u}(k')\left[\cancel{\hat{k} - \hat{k}'} - (\hat{k}-\hat{k}')\gamma^5\right] u(k) =$$

$$= m_\psi\, \bar{u}(k')\gamma^5 u(k)$$

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С другой стороны: Можно его представить как маленький член

Результат - конечен: $\langle A_\mu(q)\, A_\nu(-q)\rangle \doteq -\frac{i}{q^2-m_A^2}\left(g^{\mu\nu}-\frac{\ }{m_A^2}\right.$

Но это - только после посадки на массовую поверхность!

Вне массовой пов-ти существует предел $q \mapsto \infty$.

$$\boxed{\begin{array}{l} \xi = 0 \Rightarrow \text{Поперечная калибровка} \\ \xi \mapsto \infty \Rightarrow \text{Унитарная калибровка} \end{array}}$$

Разная теория возмущений для разных $\xi$

$\Downarrow$

В различных калибровках - перенормируемая или нет теория возмущений

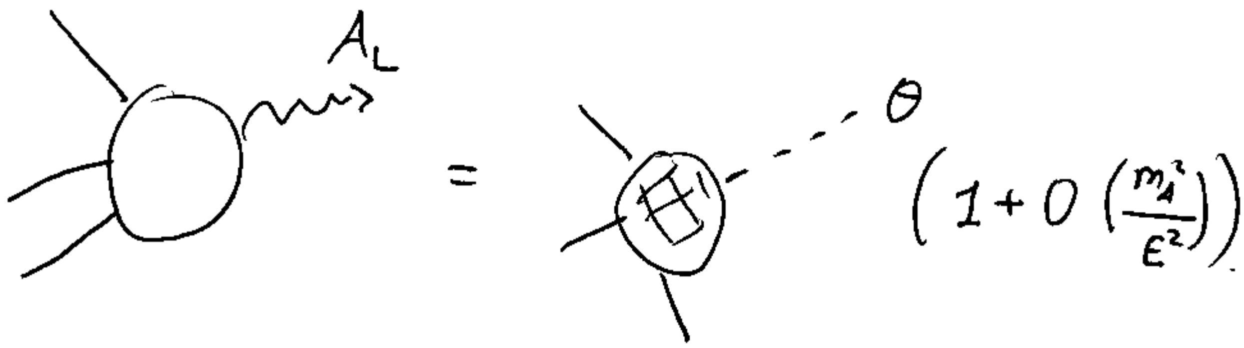
$\Downarrow$

$\xi$ - gauges $\Leftrightarrow$ Renormalizable gauges

$\Downarrow$

Сложно доказать перенормируемость в унитарной калибровке!

# Теорема эквивалентности Голдстоуновских бозонов

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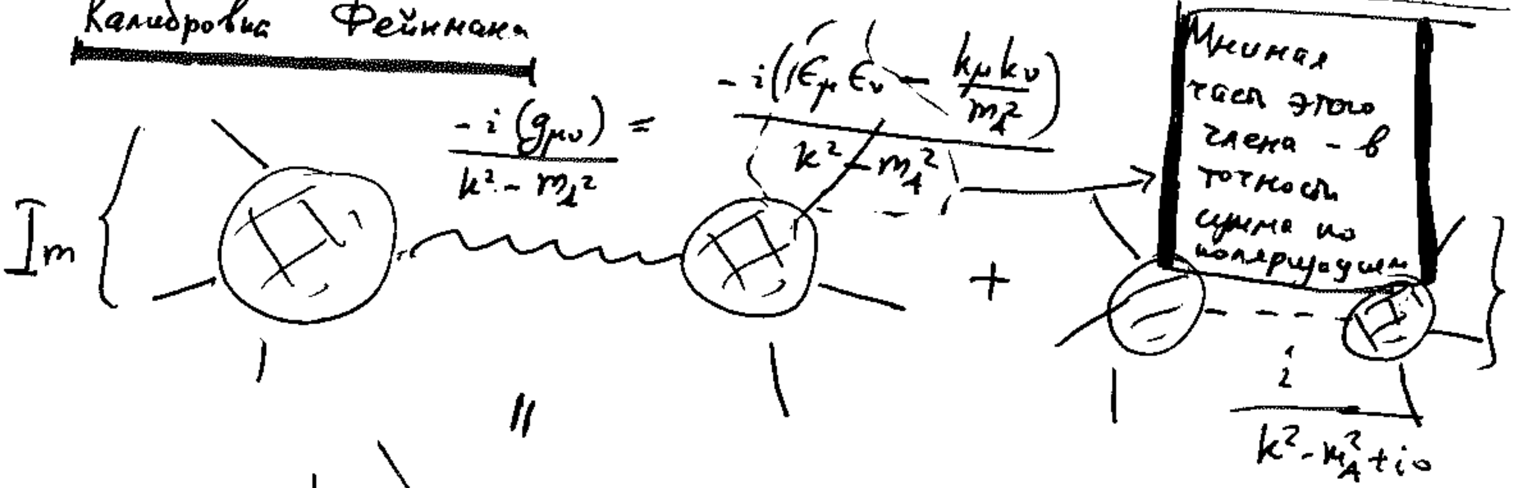
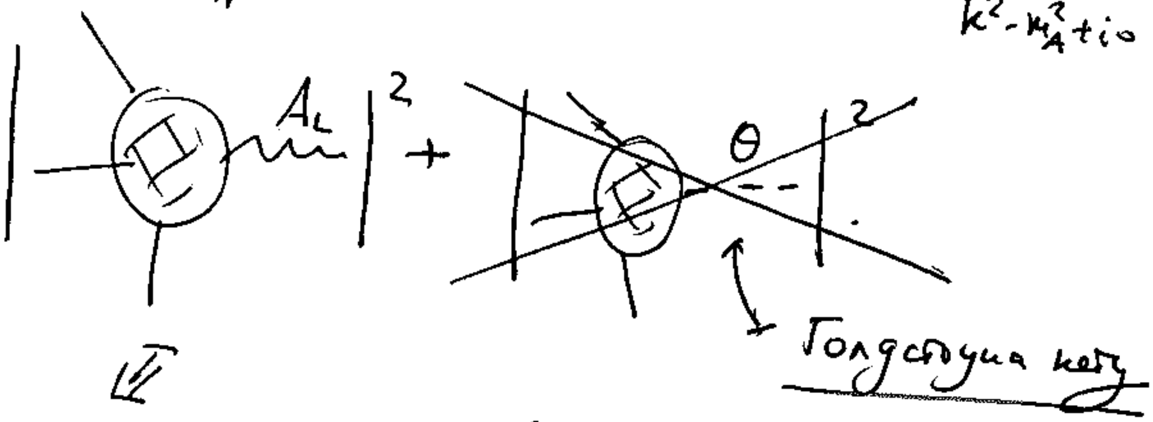
## Калибровка Фейнмана




$$-i\left(+i\pi\,\cancel{\delta(k^2-m_A^2)}\right)\frac{k_\mu k_\nu}{m_A^2}\Gamma_\mu\Gamma_\nu^* +$$

$$+\, i\left(+i\pi\,\cancel{\delta(k^2-m_A^2)}\right)\Gamma\Gamma^* = 0$$

$$\boxed{\left|\frac{k_\mu}{m_A}\Gamma_\mu\right|^2 = |\Gamma|^2}\Bigg|_{k^2=m_A^2}$$

Теперь - Кинематика:

$$v=0: \begin{cases} k_\mu = (m_A, 0, 0, 0) \\ \epsilon_\mu = \begin{matrix} (0, 1\,0\,0) \\ (0\,0\,1\,0) \\ (0\,0\,0\,1) \end{matrix} \end{cases}$$

$$\boxed{v \mapsto c} \quad \begin{cases} k_\mu = (E_k, 0, 0, k) \\ \epsilon_{\mu_L} = \left(\frac{k}{m_A}, 0, 0, \frac{E_k}{m_A}\right) \end{cases}$$

$$\boxed{\begin{matrix} \epsilon_\mu k_\mu = 0 \\ \epsilon_\mu^2 = -1 \end{matrix}}$$

Но: При $k \mapsto \infty$

$$\epsilon_{\mu_L} = \frac{k}{m_A}\left(1, 0, 0, 1\,\frac{E_k}{k}\right)$$

$$\frac{E_k}{k} \to 1 + O\left(\frac{m_A^2}{E^2}\right)$$

$$\Downarrow \quad = \frac{k_\mu}{m_A}\left(1 + O\left(\frac{m_A^2}{E^2}\right)\right).$$

$$\boxed{\frac{k_\mu}{m_A} \mapsto \epsilon_\mu}$$

$$\boxed{\left|\epsilon_\mu \Gamma_\mu\right|^2 = |\Gamma|}$$ $\Rightarrow$ Амплитуды равны по модулю.